CoastEd

Free Practice Worked Solutions

with Questions

2027 Edition

Written and taught by **Leonard Shen**

TOPICS

1 Function Basics
2 Transformations
3 Polynomials
4 Coordinate Geometry
5 Exponentials and Logarithms
6 Circular Functions
7 Composite and Inverse Functions
8 Curve Sketching
9 Differentiation
10 Integration
11 Probability and Statistics
12 Miscellaneous

USING THIS SET

These are the worked solutions to the **Free Practice Workbook**, in the same order with the same question numbers, with **each question printed above its solution**. **Workbook p.** beside each question is the page it sits on in the workbook, and each question shows the section of the **Notes** it practises.

M1 is a method mark and **A1** is an answer mark. **The trick**, **Watch out** and **Another way** and **Ways to do it** are explained in "How to use this workbook". Read them even when you got the question right: the more methods you know, the easier it is to check an answer.

Attempt the question properly before you look, and check your *method* as well as the final answer.

Calculator boxes show the TI-Nspire screen: what to type on the left, what comes back on the right. Write the equation or integral you are solving first (that is where the method mark is), and start a new problem on the calculator for every question.

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CAS commands used in these solutions

Every menu path starts from a Calculator page, and any command can also be typed letter by letter. The full CAS Guide is at the back of the Notes (Topic 13).

Command	Where to find it	Example	See notes
Actions and algebra			
Define or :=	menu 1 1 or ctrl + templates key	Define $f(x) = x^2 - 3 \cdot x$	1.3
DelVar (delete a variable)	menu 1 3	DelVar a	13.8
solve	menu 3 1	$\text{solve}(x^2 = 3 \cdot x + 4, x)$	everywhere
factor	menu 3 2	$\text{factor}(x^3 - 7 \cdot x + 6)$	3.9
expand	menu 3 3	$\text{expand}((2 \cdot x - 1)^4)$	3.7
completeSquare	menu 3 5	$\text{completeSquare}(x^2 - 2 \cdot x - 3, x)$	3.2
polyCoeffs	menu 3 8 7	$\text{polyCoeffs}(f(x), x)$	3.11
propFrac	menu 3 9 1	$\text{propFrac}\left(\frac{2 \cdot x + 7}{x + 3}, x\right)$	3.10
Calculus			
Derivative template	menu 4 1 or shift −	$\frac{d}{dx}\left(f(x)\right)$	9.2
Integral template	menu 4 3 or shift +	$\int_0^3 f(x)\,dx$	10.3
limit	menu 4 4	$\lim\left(\frac{1}{x}, x, \infty\right)$	8.2
fMin, fMax	menu 4 7, menu 4 8	$\text{fMax}(f(x), x, a, b)$	see below
tangentLine	menu 4 9	$\text{tangentLine}(x^2, x, 1)$	9.8
normalLine	menu 4 A	$\text{normalLine}(x^2, x, 1)$	9.8
Probability and statistics			
Binomial Pdf	menu 5 5 A	$\text{binomPdf}(n, p, x)$	11.6
Binomial Cdf	menu 5 5 B	$\text{binomCdf}(n, p, lower, upper)$	11.6
Normal Cdf	menu 5 5 2	$\text{normCdf}(lower, upper, \mu, \sigma)$	11.8
Inverse Normal	menu 5 5 3	$\text{invNorm}(area, \mu, \sigma)$	11.8
1-Prop z Interval	menu 6 6 5	$\text{zInterval_1Prop}\ x, n, 0.95$	11.9

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1 Function Basics

Section A – Technology free

Notes for this section: Notes 1.1–1.9

Question 1A.1* (6 marks) Maximal domains Workbook p. 5

State the maximal domain of each function. [Notes link: **1.3** Finding the maximal domain]

a. $f(x) = \sqrt{6-3x}$ 1 mark

$6-3x \geq 0 \Rightarrow x \leq 2$. **A1**
Answer a. $(-\infty, 2]$
Another way: $\sqrt{6-3x} = \sqrt{-3(x-2)}$ is $y = \sqrt{x}$ reflected in the y-axis and translated 2 right, so the endpoint is at $x = 2$ and the graph goes left. Or on CAS: $\text{domain}\left(\sqrt{6-3\cdot x}, x\right)$.

b. $g(x) = \log_e(2x+5)$ 1 mark

$2x+5 > 0 \Rightarrow x > -\frac{5}{2}$. **A1**
Answer b. $\left(-\frac{5}{2}, \infty\right)$
Another way: $y = \log_e(2x+5) = \log_e\left(2(x+\frac{5}{2})\right)$ has its asymptote moved to $x = -\frac{5}{2}$; or $\text{domain}(\ln(2\cdot x+5), x)$ on CAS.

c. $h(x) = \dfrac{3}{x^2-4}$ 1 mark

$x^2 - 4 \neq 0 \Rightarrow x \neq \pm 2$. **A1**
Answer c. $R \setminus \{-2, 2\}$

d. $k(x) = \tan(2x)$ 1 mark

$\cos(2x) \neq 0 \Rightarrow 2x \neq \frac{\pi}{2} + n\pi \Rightarrow x \neq \frac{\pi}{4} + \frac{n\pi}{2}$. **A1**
Answer d. $R \setminus \left\{\frac{\pi}{4} + \frac{n\pi}{2} : n \in Z\right\}$
Another way: sketch $y = \tan(2x)$: its period is $\frac{\pi}{2}$, so the asymptotes are at $x = \pm\frac{\pi}{4}$ and then every $\frac{\pi}{2}$.
Watch out: divide the $n\pi$ by 2 as well. The asymptotes of $\tan(2x)$ are $\frac{\pi}{2}$ apart, not π apart.

e. $p(x) = \sqrt{x+1} + \log_e(3-x)$ 2 marks

Root: $x \geq -1$. Log: $x < 3$. **M1**
Intersection: $[-1, 3)$. **A1**
Answer e. $[-1, 3)$
Another way: draw both conditions on a number line and take the overlap.
Watch out: square bracket at -1 (since $\sqrt{0} = 0$ is fine), round at 3 (since $\log_e 0$ is undefined).

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Question 1A.2** (3 marks) Odd and even functions Workbook p. 6

[Notes link: **1.7** Odd and even functions]

a. Show that $f(x) = x^4 - 3x^2$ is an even function. 1 mark

$f(-x) = (-x)^4 - 3(-x)^2 = x^4 - 3x^2 = f(x)$, so f is even. **A1**
Answer a. $f(-x) = f(x)$
Another way: every power of x is even, so the graph is symmetric about the y-axis. You can also check by graphing on CAS.

b. Show that $g(x) = \dfrac{x}{x^2+1}$ is an odd function. 1 mark

$g(-x) = \dfrac{-x}{(-x)^2+1} = -\dfrac{x}{x^2+1} = -g(x)$, so g is odd. **A1**
Answer b. $g(-x) = -g(x)$

c. Is $h(x) = x^3 + 1$ odd, even or neither? Justify your answer. 1 mark

$h(-1) = 0$, $h(1) = 2$ and $-h(1) = -2$. Since $h(-1) \neq h(1)$ and $h(-1) \neq -h(1)$, h is neither. **A1**
Answer c. Neither, e.g. $h(-1) = 0$ but $h(1) = 2$
Another way: h is $y = x^3$ translated 1 up. That moves the point of symmetry from $(0,0)$ to $(0,1)$, so it's no longer symmetric about the origin.

Question 1A.3*** (4 marks) Equal functions Workbook p. 6

Two functions are **equal** only if they have the same rule *and* the same domain. [Notes link: **1.2** Writing a function in full: domain, codomain and range]

a. Let $f(x) = \dfrac{x^2-4}{x-2}$ and $g(x) = x+2$, each with its maximal domain. Explain why f and g are not equal, and state the range of f. 2 marks

$f(x) = \frac{(x-2)(x+2)}{x-2} = x+2$, but only for $x \neq 2$: $\text{dom}\, f = R \setminus \{2\}$ while $\text{dom}\, g = R$. **A1**
The graph of f is the line $y = x+2$ with a hole at $(2,4)$, so the range is $R \setminus \{4\}$. **A1**
Answer a. Different domains; $\text{ran}\, f = R \setminus \{4\}$
Watch out: cancelling a factor doesn't put the missing point back. The domain is always read from the *original* rule.

b. Let $p(x) = \log_e(x^2)$ and $q(x) = 2\log_e(x)$, each with its maximal domain. State both domains, and the largest set on which $p(x) = q(x)$. 2 marks

$\text{dom}\, p = R \setminus \{0\}$ ($x^2 > 0$ whenever $x \neq 0$); $\text{dom}\, q = (0, \infty)$. **A1**
They agree where both are defined: $(0, \infty)$. **A1**
Answer b. $\text{dom}\, p = R \setminus \{0\}$, $\text{dom}\, q = (0, \infty)$; equal on $(0, \infty)$
Ways to do it: (1) Compare the domains from the rules, as above. (2) Sketch both: p has two branches (it's even), q has only the right-hand one. (3) On the CAS, $\text{domain}(\ln(x^2), x)$ and $\text{domain}(2 \cdot \ln(x), x)$.

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Question 1A.4**** (5 marks) Harder maximal domains Workbook p. 7

Find the maximal domain of each function. [Notes link: **1.3** Finding the maximal domain]

a. $f(x) = \log_e\left(\dfrac{x+1}{3-x}\right)$ 2 marks

Need $\frac{x+1}{3-x} > 0$: top and bottom have the same sign. **M1**
Both positive: $x > -1$ and $x < 3$. Both negative: $x < -1$ and $x > 3$, impossible. So $-1 < x < 3$. **A1**
Answer a. $(-1, 3)$
Ways to do it: (1) Sign cases, as above. (2) Multiply by $(3-x)^2 > 0$: $(x+1)(3-x) > 0$, an upside-down parabola that is positive between its roots. (3) Sign diagram with critical values -1 and 3. (4) CAS: $\text{domain}\left(\ln\left(\frac{x+1}{3-x}\right), x\right)$.
Watch out: don't multiply both sides by $(3-x)$: you don't know its sign, so the inequality might flip.

b. $g(x) = \sqrt{2-\sqrt{x}}$ 1 mark

Inner root: $x \geq 0$. Outer root: $2 - \sqrt{x} \geq 0 \Rightarrow \sqrt{x} \leq 2 \Rightarrow x \leq 4$. **A1**
Answer b. $[0, 4]$

c. $h(x) = \dfrac{\sqrt{x+2}}{\log_e(x)}$ 2 marks

Root: $x \geq -2$. Log: $x > 0$. Denominator: $\log_e(x) \neq 0 \Rightarrow x \neq 1$. **M1**
Intersection: $(0, 1) \cup (1, \infty)$. **A1**
Answer c. $(0, 1) \cup (1, \infty)$
Watch out: three conditions here, not two. A log on the bottom of a fraction gives both "inside > 0" and "log $\neq 0$".

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Section B – Technology active

Notes for this section: Notes 1.1–1.9

Question 1B.1*** (6 marks) A hybrid model Workbook p. 8

The depth of water, D metres, in a reservoir t hours after midnight is modelled by

$$D(t) = \begin{cases} 2 + \frac{1}{2}t, & 0 \le t < 4 \\ a(t-6)^2 + 3, & 4 \le t \le 10 \end{cases}$$

where a is a real constant and D is continuous over $[0, 10]$. [Notes link: **1.6** Piecewise (hybrid) functions]

a. Find the value of a. 2 marks

Continuity at $t = 4$: $2 + \frac{1}{2}(4) = a(4-6)^2 + 3$ **M1**
$4 = 4a + 3 \Rightarrow a = \frac{1}{4}$. **A1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(2 + \frac{1}{2} \cdot 4 = a \cdot (4-6)^2 + 3, a\right)$ $a = \frac{1}{4}$

Answer a. $a = \frac{1}{4}$

b. State the range of D. 2 marks

First piece: $[2, 4)$. Second piece: minimum $D(6) = 3$, maximum $D(10) = 7$, so $[3, 7]$. **M1**
Union: $[2, 7]$. **A1**
Answer b. $[2, 7]$

c. Find all times when the depth is 5 metres. Give your answers correct to two decimal places. 2 marks

First piece: $2 + \frac{1}{2}t = 5 \Rightarrow t = 6 \notin [0, 4)$, so there's no solution there.
Second piece: solve $\frac{1}{4}(t-6)^2 + 3 = 5$ for $t \in [4, 10]$. **M1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(\frac{1}{4} \cdot (t-6)^2 + 3 = 5, t\right) \mid 4 \le t \le 10$ $t = 8.828\ldots$

$t \approx 8.83$ hours. **A1**
Answer c. $t \approx 8.83$
Another way: graph D and $y = 5$ on CAS and use menu 6 4 Intersection, or solve by hand: $(t-6)^2 = 8 \Rightarrow t = 6 + 2\sqrt{2}$.
Watch out: without the domain restriction the CAS also gives $t \approx 3.17$, which is outside $[4, 10]$. Always restrict to the piece's own interval.

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Question 1B.2**** (5 marks) Domains with a parameter Workbook p. 9

Let $f(x) = \log_e(x^2 - 2kx + 9)$, where k is a real constant. [Notes link: **1.3** Finding the maximal domain]

a. Find the values of k for which the maximal domain of f is R. 2 marks

Need $x^2 - 2kx + 9 > 0$ for all x, i.e. no real roots: $\Delta = 4k^2 - 36 < 0$. **M1**
$k^2 < 9 \Rightarrow -3 < k < 3$. **A1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}((-2\cdot k)^2 - 4\cdot 1\cdot 9 < 0, k)$ $\quad -3 < k < 3$

Answer a. $k \in (-3, 3)$
Another way: complete the square: $x^2 - 2kx + 9 = (x-k)^2 + 9 - k^2$. The minimum value $9 - k^2$ must be positive, so $k^2 < 9$.
Watch out: $\Delta = 0$ isn't allowed: the quadratic would equal 0 at one point, and $\log_e(0)$ is undefined.

b. Find the maximal domain of f when $k = 5$. 2 marks

$x^2 - 10x + 9 > 0 \Rightarrow (x-1)(x-9) > 0$ **M1**
$\Rightarrow x < 1$ or $x > 9$. **A1**

TI-Nspire | Calculator page, domain (catalog)

$\text{domain}(\ln(x^2 - 10\cdot x + 9), x)$ $\quad x < 1 \text{ or } x > 9$

Answer b. $(-\infty, 1) \cup (9, \infty)$

c. Find the maximal domain of f when $k = 3$. 1 mark

$x^2 - 6x + 9 = (x-3)^2 > 0$ for all $x \neq 3$. **A1**

TI-Nspire | Calculator page, domain (catalog)

$\text{domain}(\ln(x^2 - 6\cdot x + 9), x)$ $\quad x \neq 3$

Answer c. $R \setminus \{3\}$

Another way: $\log_e\left((x-3)^2\right)$ only fails where $(x-3)^2 = 0$, i.e. $x = 3$.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 1.1–1.9

Question 1C.1** (1 mark) The base graph library Workbook p. 10

A graph has asymptotes $x = 0$ and $y = 0$ and lies entirely above the x-axis. Its rule could be

A. $y = \dfrac{1}{x}$ **B.** $y = \dfrac{1}{x^2}$

C. $y = \sqrt{x}$ **D.** $y = -\dfrac{1}{x}$

[Notes link: **1.8** The base graph library]
The truncus $y = \frac{1}{x^2}$ is positive on both sides of $x = 0$. The hyperbolas in A and D go below the axis on one side. **A1**
Answer B

Question 1C.2*** (1 mark) Domain of a sum Workbook p. 10

Let $f(x) = \sqrt{x+3}$ and $g(x) = \log_e(2-x)$. The maximal domain of $f+g$ is

A. $[-3, 2]$ **B.** $(-3, 2)$

C. $[-3, 2)$ **D.** $[-3, \infty)$

[Notes link: **1.3** Finding the maximal domain]
$\text{dom}(f+g) = \text{dom}\, f \cap \text{dom}\, g = [-3, \infty) \cap (-\infty, 2)$. **A1**
Answer C

Question 1C.3**** (1 mark) Number of solutions of a hybrid Workbook p. 10

Let $f(x) = \begin{cases} x^2, & x \leq 1 \\ 3-x, & x > 1 \end{cases}$. The equation $f(x) = p$ has exactly two solutions when

A. $0 < p < 1$ **B.** $p = 0$ or $1 \leq p < 2$

C. $p = 0$ or $1 < p < 2$ **D.** $p \geq 2$

[Notes link: **1.6** Piecewise (hybrid) functions]
Sketch it. The parabola piece gives two solutions for $0 < p \leq 1$ and one for $p = 0$ or $p > 1$. The line piece gives one solution for every $p < 2$. So: $p < 0$ one; $p = 0$ two; $0 < p \leq 1$ three; $1 < p < 2$ two; $p \geq 2$ one.
Watch out: B is the trap: at $p = 1$ the parabola gives $x = \pm 1$ and the line gives $x = 2$, three solutions. Check each boundary value.
A1
Answer C

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2 Transformations

Section A – Technology free

Notes for this section: Notes 2.1–2.8

Question 2A.1* (4 marks) Describing transformations by inspection Workbook p. 12

Describe a sequence of transformations that maps the graph of the first function onto the graph of the second. [Notes link: **2.4** The inspection method (use this one)]

a. $y = x^2$ onto $y = 3(x-2)^2+1$ 1 mark

Inspection form $y = 3(x-2)^2+1$: **A1**
Answer a. Dilation by factor 3 from the x-axis, then translation 2 right and 1 up
Another way: dash method. Write $y' = 3(x'-2)^2+1$ and compare with $y = x^2$: $x = x'-2$ and $y = \frac{y'-1}{3}$, so $x' = x+2$ and $y' = 3y+1$. Read the steps from the mapping $(x,y) \to (x+2,\ 3y+1)$.

b. $y = \sqrt{x}$ onto $y = -2\sqrt{x+1}$ 1 mark

A1
Answer b. Dilation by factor 2 from the x-axis, reflection in the x-axis, translation 1 left

c. $y = \dfrac{1}{x}$ onto $y = \dfrac{1}{3x} - 4$ 1 mark

$\frac{1}{3x} = \frac{1}{(3x)}$: the x has been replaced by $3x$. **A1**
Answer c. Dilation by factor $\frac{1}{3}$ from the y-axis, translation 4 down
Watch out: for $\frac{1}{x}$ this could also be read as a dilation by factor $\frac{1}{3}$ from the x-axis, since $\frac{1}{3x} = \frac{1}{3} \cdot \frac{1}{x}$. Both are correct.

d. $y = \log_e x$ onto $y = \log_e(2-x)$ 1 mark

$\log_e(2-x) = \log_e\big(-(x-2)\big)$: take out the negative first. **A1**
Answer d. Reflection in the y-axis, then translation 2 right
Another way: translate 2 *left* first, giving $\log_e(x+2)$, then reflect in the y-axis, giving $\log_e(-x+2)$. That's also correct: changing the order changes the translation.
Watch out: "translate 2 left" is the most common wrong answer. You have to factor out the -1 before reading the translation.

Question 2A.2*** (4 marks) Changing the order with mapping form Workbook p. 13

Let $T(x,y) = (2x+3,\ 3y-6)$. [Notes link: **2.3** The order of transformations: DRT]

a. Describe T as a sequence of transformations with the dilations first. 1 mark

Read each coordinate as "multiply, then add". **A1**
Answer a. Dilate by 2 from the y-axis and by 3 from the x-axis, then translate 3 right and 6 down

b. By factorising each coordinate of T, describe T with the translations first. 2 marks

$2x+3 = 2\left(x+\frac{3}{2}\right)$ and $3y-6 = 3(y-2)$ **M1**
Now it's "add, then multiply". **A1**
Answer b. Translate $\frac{3}{2}$ right and 2 down, then dilate by 2 from the y-axis and by 3 from the x-axis
Watch out: "translate 3 right, then dilate" is wrong: the dilation would stretch the translation too, giving $2x+6$.

c. The graph of $y = f(x)$ is translated 1 unit left, then dilated by a factor of 3 from the y-axis. By writing this in mapping form and expanding, describe the same transformation with the dilation first. 1 mark

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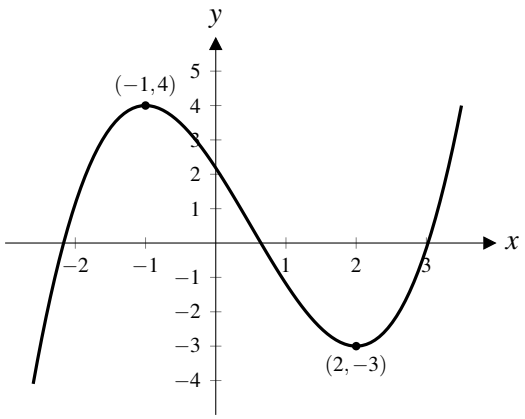
$x \to x-1 \to 3(x-1) = 3x-3$ **A1**
Answer c. Dilate by 3 from the y-axis, then translate 3 left

Question 2A.3** (3 marks) Vertical translations and the number of solutions Workbook p. 14

The graph of a cubic $y = f(x)$ is shown below. Its turning points are $(-1,4)$ and $(2,-3)$. [Notes link: **2.7** Sketching using transformations]



Find the values of k for which the equation $f(x)+k=0$ has

a. exactly three solutions 1 mark

The graph of $y = f(x)+k$ is the graph translated k up, with turning points $(-1,4+k)$ and $(2,-3+k)$. Three solutions when the maximum is above the axis and the minimum is below: $4+k>0$ and $-3+k<0$. **A1**
Answer a. $-4<k<3$
Another way: $f(x)+k=0 \Leftrightarrow f(x)=-k$: count where the horizontal line $y=-k$ cuts the graph. Three cuts for $-3<-k<4$.

b. exactly two solutions 1 mark

A turning point sits on the x-axis. **A1**
Answer b. $k=-4$ or $k=3$

c. exactly one solution 1 mark

Both turning points on the same side of the axis. **A1**
Answer c. $k<-4$ or $k>3$
Watch out: translating up by k adds k: the maximum moves to $4+k$, not $4-k$.

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Question 2A.4**** (5 marks) Finding a transformation from two points Workbook p. 15

The transformation $T(x,y) = (ax+b,\ cy+d)$ maps $(1,2)$ onto $(5,1)$ and $(3,6)$ onto $(9,-7)$. [Notes link: **2.5** The mapping (dash) method]

a. Find the values of a, b, c and d. 2 marks

x: $a+b=5$ and $3a+b=9 \Rightarrow a=2,\ b=3$. **A1**
y: $2c+d=1$ and $6c+d=-7 \Rightarrow c=-2,\ d=5$. **A1**
Answer a. $a=2,\ b=3,\ c=-2,\ d=5$

b. Both points lie on the line $y=2x$. Find the equation of the image of this line under T. 2 marks

$x'=2x+3 \Rightarrow x=\frac{x'-3}{2};\ y'=-2y+5 \Rightarrow y=\frac{5-y'}{2}$. **M1**
$\frac{5-y'}{2}=2\cdot\frac{x'-3}{2} \Rightarrow y=11-2x$ **A1**
Answer b. $y=11-2x$
Ways to do it: (1) Dash method, as above. (2) The image of a line is a line, so just find the line through the two image points $(5,1)$ and $(9,-7)$: gradient -2, so $y-1=-2(x-5)$. (3) Check either answer with a third point: $(0,0)\to(3,5)$, and $11-6=5$. ✓

c. Find the coordinates of the point that T maps onto itself. 1 mark

$x=2x+3 \Rightarrow x=-3;\ y=-2y+5 \Rightarrow y=\frac{5}{3}$ **A1**
Answer c. $\left(-3,\frac{5}{3}\right)$

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Section B – Technology active

Notes for this section: Notes 2.1–2.8

Question 2B.1*** (4 marks) Transforming a function with a restricted domain Workbook p. 16

Let $f:[-1,3]\to R,\ f(x)=x^2-2x$. The graph of $y=f(x)$ is transformed by $T(x,y)=(1-2x,\ 3y+2)$. [Notes link: **2.5** The mapping (dash) method]

a. Find the rule of the image in the form $y=3f(\ldots)+2$, and its domain. 2 marks

$x'=1-2x\Rightarrow x=\frac{1-x'}{2};\ y'=3y+2\Rightarrow y=\frac{y'-2}{3}$. So $y=3f\left(\frac{1-x}{2}\right)+2$. **M1**
Define f *with its domain*, and the CAS gives the image's domain automatically:

TI-Nspire | Calculator page

Define $f(x)=x^2-2\cdot x\,|\,-1\le x\le 3$

$3\cdot f\left(\frac{1-x}{2}\right)+2$ $\qquad$ $\frac{3x^2}{4}+\frac{3x}{2}-\frac{1}{4}\,|\,-5\le x\le 3$

A1

Answer a. $y=3f\left(\frac{1-x}{2}\right)+2=\frac{3}{4}x^2+\frac{3}{2}x-\frac{1}{4}$, domain $[-5,3]$
Another way: map the endpoints by hand: $x=-1\to 1-2(-1)=3$ and $x=3\to 1-6=-5$. The reflection flips the interval, giving $[-5,3]$.

b. State the range of the image. 2 marks

Range of f: minimum $f(1)=-1$, maximum $f(-1)=f(3)=3$, so $[-1,3]$. **M1**
Apply $y\to 3y+2$: $[-1,11]$. **A1**
Answer b. $[-1,11]$
Another way: graph the image on CAS with its domain and read off the lowest and highest points.

Question 2B.2***** (3 marks) Sliding a cubic until it touches the axis Workbook p. 16

Let $f:R\to R,\ f(x)=x(x-3)(x-8)$ and $g:R\to R,\ g(x)=(x-a)^2(x-b)$, where $g(x)=f(x)+k$ and $a,b,k\in R$. [Notes link: **2.7** Sketching using transformations]

Find all possible values of a and b. 3 marks

The trick: a repeated factor means the graph touches the axis, so g is f slid vertically until a turning point sits on the x-axis.
g is f translated k units vertically, and the repeated factor $(x-a)^2$ means the graph of g **touches** the x-axis at $x=a$. So a turning point of f has been moved onto the x-axis. There are two turning points, so there are two answers. **M1**
Equate coefficients: $(x-a)^2(x-b)=x^3-(2a+b)x^2+\left(a^2+2ab\right)x-a^2b$ and $f(x)+k=x^3-11x^2+24x+k$.

TI-Nspire | Calculator page, menu 3 1 solve

solve$(2\cdot a+b=11$ and $a^2+2\cdot a\cdot b=24,\{a,b\})$ $\qquad$ $a=\frac{4}{3}$ and $b=\frac{25}{3}$ or $a=6$ and $b=-1$

M1 A1

Answer $a=6,\ b=-1$ or $a=\frac{4}{3},\ b=\frac{25}{3}$
Ways to do it: (1) Equate coefficients and solve on the CAS, as above. The constant terms give $k=-a^2b$: $k=36$ or $k=-\frac{400}{27}$. (2) By hand: $b=11-2a$, so $a^2+2a(11-2a)=24\Rightarrow 3a^2-22a+24=0\Rightarrow(3a-4)(a-6)=0$. (3) Graph page: graph f, find the turning points with menu 6 (Analyse Graph): $\left(\frac{4}{3},\frac{400}{27}\right)$ and $(6,-36)$. a is the x-value of a turning point and k is the opposite of its y-value. Then solve $f(x)+k=0$ for the other intercept b. (4) Slider: graph $f(x)+k$ with a slider for k and watch for the two values where the graph touches the axis. (5) Once you know calculus (Topic 9): solve $f'(x)=0$ for a.
Watch out: there are two answers. One turning point is slid down onto the axis and the other is slid up.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 2.1–2.8

Question 2C.1*** (1 mark) Domain and range of an image Workbook p. 17

The function f has domain $[-2,4]$ and range $[0,3]$. The function $g(x)=1-f\left(\frac{x}{2}\right)$ has

A. domain $[-1,2]$ and range $[-2,1]$

B. domain $[-4,8]$ and range $[-2,1]$

C. domain $[-4,8]$ and range $[1,4]$

D. domain $[-4,8]$ and range $[-3,0]$

[Notes link: **2.7** Sketching using transformations]
Dilation by 2 from the y-axis doubles the domain: $[-4,8]$. Reflect the range in the x-axis: $[-3,0]$, then up 1: $[-2,1]$. **A1**
Answer B

Question 2C.2** (1 mark) Transforming a relation Workbook p. 17

The circle $x^2+y^2=4$ is translated 3 units right and 1 unit down. The image has equation

A. $(x+3)^2+(y-1)^2=4$

B. $(x-3)^2+(y-1)^2=4$

C. $(x-3)^2+(y+1)^2=4$

D. $(x+3)^2+(y+1)^2=4$

[Notes link: **2.2** What happens to the equation: the opposite]
Replace x with $x-3$ and y with $y+1$: always the opposite, for y as well. **A1**
Answer C

Question 2C.3**** (1 mark) Working back from the image Workbook p. 17

The transformation $(x,y)\to\left(\frac{x}{2}+1,\ -y\right)$ maps the graph of $y=f(x)$ onto a graph that passes through $(4,3)$. Which point must lie on the graph of $y=f(x)$?

A. $(3,-3)$

B. $(6,3)$

C. $(6,-3)$

D. $(10,-3)$

[Notes link: **2.8** From a sequence of transformations to the equation]
Undo the mapping: $\frac{x}{2}+1=4\Rightarrow x=6$ and $-y=3\Rightarrow y=-3$.
Watch out: A applies the mapping forwards to $(4,3)$, which answers a different question.
A1
Answer C

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3 Polynomials

Section A – Technology free

Notes for this section: Notes 3.1–3.11

Question 3A.1* (5 marks) Quadratics Workbook p. 19

[Notes link: **3.2** Quadratics]

a. Solve $2x^2-5x-3=0$. 1 mark

$(2x+1)(x-3)=0$ **A1**
Answer a. $x=-\frac{1}{2}$ or $x=3$
Another way: quadratic formula: $x=\frac{5\pm\sqrt{25+24}}{4}=\frac{5\pm7}{4}$.

b. Solve $x^2-4x-1=0$, giving exact answers. 2 marks

Complete the square: $(x-2)^2-5=0$ **M1**
$x-2=\pm\sqrt{5}$ **A1**
Answer b. $x=2\pm\sqrt{5}$
Another way: quadratic formula: $x=\frac{4\pm\sqrt{16+4}}{2}=2\pm\sqrt{5}$.

c. Find the values of k for which $x^2+kx+9=0$ has two distinct real solutions. [Notes link: **3.2** Taking the formula apart: the discriminant] 2 marks

Two solutions $\Rightarrow\Delta>0$: $k^2-4(1)(9)>0$ **M1**
$k^2>36\Rightarrow k<-6$ or $k>6$ **A1**
Answer c. $k\in(-\infty,-6)\cup(6,\infty)$
Watch out: $k^2>36$ is *not* $k>\pm6$. Sketch $y=k^2-36$ and read off where it's positive.

Question 3A.2** (4 marks) Division into inspection form Workbook p. 20

Let $f(x)=\dfrac{2x+7}{x+3}$. [Notes link: **3.10** Division into inspection form]

a. Write $f(x)$ in the form $a+\dfrac{b}{x+c}$. 2 marks

$\dfrac{2x+7}{x+3}=\dfrac{2(x+3)+1}{x+3}$ **M1**
$=2+\dfrac{1}{x+3}$ **A1**
Answer a. $f(x)=2+\dfrac{1}{x+3}$
Another way: long division of $2x+7$ by $x+3$: quotient 2, remainder 1.

b. State the equations of the asymptotes and the axis intercepts of $y=f(x)$. 2 marks

Asymptotes $x=-3$, $y=2$. **A1**
x-intercept: $2x+7=0\Rightarrow x=-\frac{7}{2}$. y-intercept: $f(0)=\frac{7}{3}$. **A1**
Answer b. Asymptotes $x=-3$, $y=2$; intercepts $\left(-\frac{7}{2},0\right)$, $\left(0,\frac{7}{3}\right)$

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Question 3A.3*** (3 marks) A cubic inequality Workbook p. 20

[Notes link: **3.3** Cubics]

Solve $x^3-4x^2+x+6\geq 0$. 3 marks

$P(-1)=-1-4-1+6=0$, so $(x+1)$ is a factor: $P(x)=(x+1)(x^2-5x+6)=(x+1)(x-2)(x-3)$ **M1**
Positive cubic crossing at -1, 2 and 3: above the axis between -1 and 2, and after 3. **M1**
$-1\leq x\leq 2$ or $x\geq 3$ **A1**
Answer $[-1,2]\cup[3,\infty)$
Ways to do it: (1) Factorise and sketch, as above. (2) Sign table: test one value in each of the four intervals. (3) CAS: $\text{solve}(x^3-4\cdot x^2+x+6\geq 0,x)$.
Watch out: "$\geq$" means the intercepts are included: square brackets.

Question 3A.4**** (3 marks) Dividing by a quadratic Workbook p. 21

[Notes link: **3.8** Dividing polynomials: the language]

Divide x^4+2x^3-x+5 by x^2+1. State the quotient and the remainder. 3 marks

Put in $0x^2$ so the columns line up, and divide by the leading term x^2 each time: **M1 M1**

$$\begin{array}{rrrrr} & & x^2 & +2x & -1 \\ \hline x^2+1\,) & x^4 & +2x^3 & +0x^2 & -x & +5 \\ & x^4 & & +x^2 & & \\ \hline & & 2x^3 & -x^2 & -x & \\ & & 2x^3 & & +2x & \\ \hline & & & -x^2 & -3x & +5 \\ & & & -x^2 & & -1 \\ \hline & & & & -3x & +6 \end{array}$$

A1

Answer Quotient x^2+2x-1, remainder $-3x+6$
Ways to do it: (1) Long division, as above. (2) Equate coefficients: write $x^4+2x^3-x+5=(x^2+1)(x^2+bx+c)+dx+e$ and expand. (3) CAS: $\text{propFrac}\left(\frac{x^4+2\cdot x^3-x+5}{x^2+1}\right)$.
Watch out: dividing by a quadratic leaves a *linear* remainder, not a constant. Stop when the degree is lower than the divisor's.

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Section B – Technology active

Notes for this section: Notes 3.1–3.11

Question 3B.1*** (3 marks) A repeated factor Workbook p. 22

[Notes link: **3.11** Equating coefficients]

$P(x) = x^3 + ax^2 + bx + 2$ has $(x-1)^2$ as a factor. Find a and b, and hence factorise $P(x)$. 3 marks

$P(x) = (x-1)^2(x+c)$ for some c (the constant term forces $c = 2$). Expand: $x^3 + (c-2)x^2 + (1-2c)x + c$. **M1**
Equate: $c = 2$, so $a = c - 2 = 0$ and $b = 1 - 2c = -3$. **M1**

TI-Nspire | Calculator page, menu 3 3 expand

$\text{expand}((x-1)^2 \cdot (x+c))$

$\text{solve}(c-2 = a \text{ and } 1-2\cdot c = b \text{ and } c = 2, a, b, c) \quad a = 0 \text{ and } b = -3 \text{ and } c = 2$

A1

Answer $a = 0$, $b = -3$; $P(x) = (x-1)^2(x+2)$
Another way: use $P(1) = 0$ and $P'(1) = 0$ (a repeated root is also a stationary point): $3 + a + b = 0$ and $3 + 2a + b = 0$.

Question 3B.2***** (4 marks) A quartic with a parameter Workbook p. 22

Let $f(x) = x^4 - 4x^3$. [Notes link: **3.4** Quartics]

a. Using technology, find the coordinates of the stationary points of the graph of f and state their nature. 2 marks

Graph f and use menu 6 Analyse Graph, or solve $f'(x) = 0$. **M1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(\frac{d}{dx}(x^4 - 4\cdot x^3) = 0, x\right) \quad x = 0 \text{ or } x = 3$

$(0,0)$ is a stationary point of inflection (the graph flattens but keeps falling); $(3,-27)$ is a local minimum. **A1**
Answer a. $(0,0)$ stationary point of inflection; $(3,-27)$ local minimum

b. Find the values of k for which the equation $x^4 - 4x^3 + k = 0$ has two solutions, both positive. 2 marks

$f(x) = -k$: a horizontal line $y = -k$ cuts the graph twice whenever $-k > -27$, and both cuts are at positive x only when the line is below the x-axis: $-27 < -k < 0$. **M1**
$0 < k < 27$ **A1**
Answer b. $0 < k < 27$
Watch out: at $k = 0$ the solutions are $x = 0$ and $x = 4$, and 0 isn't positive. For $k < 0$ one solution is negative.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 3.1–3.11

Question 3C.1** (1 mark) Factor theorem with an unknown Workbook p. 23

$x-2$ is a factor of x^3+ax^2-4x+4. The value of a is

A. -2 **B.** -1

C. 0 **D.** 1

[Notes link: **3.9** Factorising polynomials: the one question to know]
$P(2)=8+4a-8+4=0 \Rightarrow a=-1$. **A1**
Answer B

Question 3C.2*** (1 mark) A quartic in disguise Workbook p. 23

The number of real solutions of $x^4-2x^2-8=0$ is

A. 0 **B.** 1

C. 2 **D.** 3

[Notes link: **3.5** How many solutions? Removing solutions]
$(x^2-4)(x^2+2)=0$. $x^2=4$ gives $x=\pm 2$; $x^2=-2$ gives nothing. **A1**
Answer C

Question 3C.3**** (1 mark) Number of roots of a cubic Workbook p. 23

The equation $x^3-3x+k=0$ has three distinct real solutions when

A. $k>2$ **B.** $k<-2$

C. $-2<k<2$ **D.** $k=\pm 2$

[Notes link: **3.3** Cubics]
The turning points of $y=x^3-3x+k$ are at $x=\pm 1$, with y-values $k+2$ and $k-2$. Three roots need one above the axis and one below: $k+2>0$ and $k-2<0$. (On the CAS, use a slider for k.) **A1**
Answer C

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4 Coordinate Geometry

Section A – Technology free

Notes for this section: Notes 4.1–4.5

Question 4A.1* (2 marks) Simultaneous equations Workbook p. 25

[Notes link: **4.4** Simultaneous linear equations]

Solve $3x-2y=12$ and $x+4y=-10$ simultaneously. 2 marks

From the second: $x=-10-4y$. Substitute: $3(-10-4y)-2y=12 \Rightarrow -14y=42 \Rightarrow y=-3$. **M1**
$x=-10+12=2$. **A1**
Answer $x=2$, $y=-3$
Another way: substitution: $x=-10-4y$ into the first equation. Or CAS: $\text{solve}(3\cdot x-2\cdot y=12 \text{ and } x+4\cdot y=-10, x, y)$.

Question 4A.2*** (3 marks) Perpendicular bisectors and equal distances Workbook p. 25

Let $A=(-1,2)$ and $B=(5,6)$. [Notes link: **4.1** Equations of lines]

a. Find the equation of the perpendicular bisector of AB. 2 marks

Midpoint $(2,4)$; gradient of AB is $\frac{4}{6}=\frac{2}{3}$, so the perpendicular gradient is $-\frac{3}{2}$. **M1**
$y-4=-\frac{3}{2}(x-2) \Rightarrow 3x+2y=14$ **A1**
Answer a. $3x+2y=14$

b. Find the point on the x-axis that is the same distance from A as from B. 1 mark

Every point on the perpendicular bisector is equidistant from A and B. Put $y=0$: $x=\frac{14}{3}$. **A1**
Answer b. $\left(\frac{14}{3},0\right)$
Another way: let the point be $(p,0)$ and solve $(p+1)^2+4=(p-5)^2+36$.

Question 4A.3**** (4 marks) The shortest distance from a point to a line Workbook p. 26

[Notes link: **4.5** General tips for coordinate geometry]

Find the shortest distance from the point $P(4,5)$ to the line $y=2x+2$, and the point on the line that is closest to P. 4 marks

The shortest path is perpendicular to the line: gradient $-\frac{1}{2}$ through P: $y-5=-\frac{1}{2}(x-4)$ **M1**
Intersect: $2x+2=7-\frac{x}{2} \Rightarrow x=2$, $y=6$ **M1 A1**
Distance from $(4,5)$ to $(2,6)$: $\sqrt{4+1}=\sqrt{5}$ **A1**
Answer $\sqrt{5}$, at the point $(2,6)$
Ways to do it: (1) Perpendicular line, as above (fastest tech-free). (2) Minimise the distance squared: $D^2=(x-4)^2+(2x+2-5)^2=5x^2-20x+25=5(x-2)^2+5$, minimum 5 at $x=2$. (3) Calculus on D^2 (Topic 9). (4) CAS: $\text{fMin}((x-4)^2+(2\cdot x-3)^2, x)$.
Watch out: minimise D^2, not D: it has the same minimiser and no square root to differentiate. Remember to take the square root at the end.

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Question 4A.4** (6 marks) Forms of a line Workbook p. 26

[Notes link: **4.1** Equations of lines]

a. A line has x-intercept 4 and y-intercept -3. Write its equation in intercept form, and in the form $ax+by=c$ where a, b and c are integers. 2 marks

Intercept form: $\dfrac{x}{4}+\dfrac{y}{-3}=1$ **A1**
Multiply through by 12: $3x-4y=12$ **A1**
Answer a. $\frac{x}{4}-\frac{y}{3}=1;\ 3x-4y=12$
Another way: gradient $=\frac{0-(-3)}{4-0}=\frac{3}{4}$, so $y=\frac{3}{4}x-3$, then rearrange.

b. Find the equation of the line through $(2,-5)$ that is (i) parallel to the line $x=7$ (ii) perpendicular to the line $x=7$. 2 marks

$x=7$ is vertical. (i) Another vertical line: $x=2$ **A1**
(ii) A horizontal line: $y=-5$ **A1**
Answer b. (i) $x=2$ (ii) $y=-5$
Watch out: a vertical line has no gradient, so $m_1m_2=-1$ can't be used here. Think about the picture instead.

c. Find the value of m for which the line $y=mx+1$ is (i) parallel to $2x-3y=6$ (ii) perpendicular to $2x-3y=6$. 2 marks

$2x-3y=6\Rightarrow y=\frac{2}{3}x-2$, gradient $\frac{2}{3}$. (i) $m=\frac{2}{3}$ **A1**
(ii) Negative reciprocal: $m=-\frac{3}{2}$ **A1**
Answer c. (i) $\frac{2}{3}$ (ii) $-\frac{3}{2}$

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Section B – Technology active

Notes for this section: Notes 4.1–4.5

Question 4B.1*** (5 marks) Extended response: a quadrilateral Workbook p. 28

The points $A(1,1)$, $B(5,3)$, $C(7,7)$ and $D(3,5)$ are the vertices of a quadrilateral. [Notes link: **4.2** The formula sheet you should already know]

a. Show that $ABCD$ is a rhombus. 2 marks

AB: gradient $\frac{2}{4}=\frac{1}{2}$, length $\sqrt{20}$. DC: gradient $\frac{1}{2}$, length $\sqrt{20}$. So $AB \parallel DC$ and $AB=DC$: a parallelogram. **M1**
$BC=\sqrt{4+16}=\sqrt{20}=AB$: adjacent sides equal, so it's a rhombus. **A1**
Answer a. Opposite sides parallel and all four sides $\sqrt{20}$

b. Show that the diagonals are perpendicular. 1 mark

Gradient $AC=\frac{6}{6}=1$; gradient $BD=\frac{2}{-2}=-1$; product -1. **A1**
Answer b. $m_{AC}m_{BD}=-1$

c. Find the area of $ABCD$. 2 marks

Area of a rhombus $=\frac{1}{2}\times$ product of diagonals: $AC=6\sqrt{2}$, $BD=2\sqrt{2}$ **M1**
$\frac{1}{2}\cdot 6\sqrt{2}\cdot 2\sqrt{2}=12$ **A1**
Answer c. 12 square units
Ways to do it: (1) Half the product of the diagonals. (2) Split into two triangles along AC: each has base $6\sqrt{2}$ and height $\sqrt{2}$. (3) Box method: draw the bounding rectangle and subtract the corner triangles.

Question 4B.2***** (6 marks) A line that just touches a circle Workbook p. 28

The line with equation $y=mx-2$, where $m>0$, can be tangent to the circle with equation

$$(x-1)^2+\left(y-\left(\sqrt{3}-2\right)\right)^2=2-\sqrt{3}$$

for two distinct values of m. [Notes link: **4.3** The angle between two lines]

a. Find the two values of m and the corresponding points of tangency with the circle. 3 marks

The trick: tangent means the line and the circle meet exactly once, so substitute and set $\Delta=0$. Every one of these lines passes through $(0,-2)$.

Sub $y=mx-2$: $(x-1)^2+\left(mx-\sqrt{3}\right)^2=2-\sqrt{3}$, i.e. $\left(m^2+1\right)x^2-2\left(1+\sqrt{3}m\right)x+2+\sqrt{3}=0$. **M1**
$\Delta=4\left(1+\sqrt{3}m\right)^2-4\left(m^2+1\right)\left(2+\sqrt{3}\right)=0$ gives $\left(\sqrt{3}-1\right)m^2-2\sqrt{3}m+\left(\sqrt{3}+1\right)=0$, so $m=1$ or $m=2+\sqrt{3}$. **A1**
Touching point: $x=\dfrac{1+\sqrt{3}m}{m^2+1}$. $m=1$: $\left(\frac{\sqrt{3}+1}{2},\ \frac{\sqrt{3}-3}{2}\right)$. $m=2+\sqrt{3}$: $\left(\frac{1}{2},\ \frac{\sqrt{3}}{2}-1\right)$. **A1**

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$\text{solve}(4\cdot(1+\sqrt{3}\cdot m)^2-4\cdot(m^2+1)\cdot(2+\sqrt{3})=0,m)$ $\qquad m=1 \text{ or } m=\sqrt{3}+2$

Answer a. $m=1$ at $\left(\frac{\sqrt{3}+1}{2},\ \frac{\sqrt{3}-3}{2}\right)$; $m=2+\sqrt{3}$ at $\left(\frac{1}{2},\ \frac{\sqrt{3}}{2}-1\right)$

Another way: the distance from the centre $\left(1,\sqrt{3}-2\right)$ to the line $mx-y-2=0$ must equal the radius: $\dfrac{|m-\sqrt{3}|}{\sqrt{m^2+1}}=\sqrt{2-\sqrt{3}}$. Squaring gives the same quadratic in m. (Distance from a point to a line is a Specialist formula.)

b. Find the area of the region bounded by the two tangent lines and the minor arc of the circle. 3 marks

Both tangents start from $P(0,-2)$. Their gradients are 1 and $2+\sqrt{3}$, so their inclinations are 45° and 75° and the angle between them is 30°. **M1**
Let C be the centre and T_1, T_2 the touching points. $PC=\sqrt{1+3}=2$, $r^2=2-\sqrt{3}$, and each tangent has length

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$\sqrt{4-\left(2-\sqrt{3}\right)}=\sqrt{2+\sqrt{3}}$. The kite PT_1CT_2 is two right-angled triangles: area $=2\times\frac{1}{2}\sqrt{2+\sqrt{3}}\sqrt{2-\sqrt{3}}=1$. **M1**

The angle at C is $180^\circ-30^\circ=150^\circ=\frac{5\pi}{6}$, so the sector has area $\frac{1}{2}r^2\cdot\frac{5\pi}{6}=\frac{5\pi\left(2-\sqrt{3}\right)}{12}$.

Area $=1-\dfrac{5\pi\left(2-\sqrt{3}\right)}{12}\approx 0.649$ **A1**

Answer b. $1-\frac{5\pi\left(2-\sqrt{3}\right)}{12}\approx 0.649$ square units

Watch out: a radius meets a tangent at 90°, which is what makes the kite two right-angled triangles. The sector is the one facing P (the minor arc), so its angle is 150°, not 210°.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 4.1–4.5

Question 4C.1*** (1 mark) Unique solution Workbook p. 30

The simultaneous equations $x+ky=2$ and $kx+4y=4$ have a unique solution when

A. $k=2$ **B.** $k \in R \setminus \{2\}$

C. $k=\pm 2$ **D.** $k \in R \setminus \{-2,2\}$

[Notes link: **4.4** Simultaneous linear equations]
Unique means the lines aren't parallel: every k except ± 2. **A1**
Answer D

Question 4C.2** (1 mark) Literal equations Workbook p. 30

If $ax+b=cx+d$, where $a \neq c$, then x equals

A. $\dfrac{d-b}{a-c}$ **B.** $\dfrac{b-d}{a-c}$

C. $\dfrac{d-b}{a+c}$ **D.** $\dfrac{d+b}{a-c}$

[Notes link: **4.5** General tips for coordinate geometry]
$ax-cx=d-b \Rightarrow x(a-c)=d-b$. **A1**
Answer A

Question 4C.3**** (1 mark) Distance between parallel lines Workbook p. 30

The distance between the parallel lines $y=2x+1$ and $y=2x+6$ is

A. 5 **B.** $\sqrt{5}$

C. $\frac{5}{2}$ **D.** $2\sqrt{5}$

[Notes link: **4.5** General tips for coordinate geometry]
Take the point $(0,1)$ on the first line. The perpendicular through it, $y=1-\frac{x}{2}$, meets the second line at $(-2,2)$. Distance $\sqrt{4+1}=\sqrt{5}$.
Watch out: A is the *vertical* gap, not the shortest distance.
A1
Answer B

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5 Exponentials and Logarithms

Section A – Technology free

Notes for this section: Notes 5.1–5.10

Question 5A.1* (3 marks) Index laws Workbook p. 32

Simplify each expression, writing your answer with positive indices. [Notes link: **5.1** Index laws]

a. $\dfrac{(8x^6y^3)^{\frac{2}{3}}}{4x^2y}$ 2 marks

$(8x^6y^3)^{\frac{2}{3}} = 8^{\frac{2}{3}}x^4y^2 = 4x^4y^2$ **M1**

$\dfrac{4x^4y^2}{4x^2y} = x^2y$ **A1**

Answer a. x^2y

Another way: write 8 as 2^3 first: $(2^3x^6y^3)^{\frac{2}{3}} = 2^2x^4y^2$.

b. $\dfrac{3^{n+2} - 3^n}{8 \times 3^n}$ 1 mark

Factor out 3^n: $\dfrac{3^n(9-1)}{8 \times 3^n} = 1$ **A1**

Answer b. 1

Question 5A.2** (3 marks) Sketching a log graph Workbook p. 32

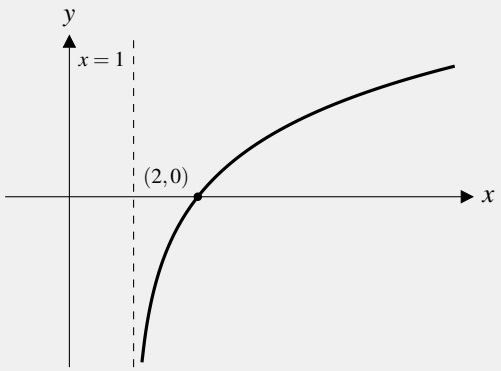
[Notes link: **5.6** Graphs: growth and decay]

Sketch $y = 2\log_e(x-1)$ on the axes below, labelling the asymptote with its equation and the x-intercept with its coordinates. 3 marks

From $y = \log_e x$: dilate by 2 from the x-axis, translate 1 right. Asymptote $x = 1$. **M1**

x-intercept: $\log_e(x-1) = 0 \Rightarrow x = 2$. **A1**

Shape: increasing, concave down, heading down the asymptote as $x \to 1^+$. **A1**



Answer Asymptote $x = 1$; x-intercept $(2, 0)$

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Question 5A.3**** (5 marks) Log equations with different bases Workbook p. 33

[Notes link: **5.5** Changing the base]

a. Solve $\log_2(x) = \log_4(x+6)$. 3 marks

Change base: $\log_4(x+6) = \dfrac{\log_2(x+6)}{\log_2 4} = \frac{1}{2}\log_2(x+6)$ **M1**
$2\log_2 x = \log_2(x+6) \Rightarrow x^2 = x+6 \Rightarrow (x-3)(x+2) = 0$ **M1**
Domain needs $x > 0$: $x = 3$. **A1**
Answer a. $x = 3$
Ways to do it: (1) Change the larger base to the smaller one, as above. (2) Let both sides equal t: $x = 2^t$ and $x+6 = 4^t = (2^t)^2 = x^2$. (3) Write everything in $\log_e$: $\frac{\ln x}{\ln 2} = \frac{\ln(x+6)}{2\ln 2}$. (4) CAS solve.

b. Solve $2\log_3(x) - \log_3(x+6) = 1$. 2 marks

$\log_3\left(\frac{x^2}{x+6}\right) = 1 \Rightarrow \frac{x^2}{x+6} = 3 \Rightarrow x^2 - 3x - 18 = 0$ **M1**
$(x-6)(x+3) = 0$; need $x > 0$, so $x = 6$. **A1**
Answer b. $x = 6$
Watch out: the 1 on the right is $\log_3 3$. A common slip is writing $\frac{x^2}{x+6} = 1$.

Question 5A.4*** (3 marks) Simultaneous index and log equations Workbook p. 33

[Notes link: **5.2** Solving exponential equations]

a. Solve $2^{x+y} = 32$ and $3^{x-y} = 27$ simultaneously. 1 mark

$x+y = 5$ and $x-y = 3$ **A1**
Answer a. $x = 4$, $y = 1$

b. Solve $\log_2(x) + \log_2(y) = 5$ and $\log_2(x) - \log_2(y) = 1$ simultaneously. 2 marks

Add: $2\log_2 x = 6 \Rightarrow x = 8$. **M1**
Subtract: $2\log_2 y = 4 \Rightarrow y = 4$. **A1**
Answer b. $x = 8$, $y = 4$
Another way: turn them into $xy = 32$ and $\frac{x}{y} = 2$ and solve those.

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Section B – Technology active

Notes for this section: Notes 5.1–5.10

Question 5B.1***** (5 marks) Two doses Workbook p. 34

The concentration of a medicine in the blood t hours after one dose is $C(t) = 10te^{-\frac{t}{2}}$ mg/L. [Notes link: **5.8** Applications: it's just transformations]

a. Using technology, find the maximum concentration from a single dose and the time at which it occurs. Give the concentration exactly. 2 marks

Graph C and use menu 6 3 Maximum, or use fMax. **M1**

TI-Nspire | Calculator page, menu 4 8 fMax

$\text{fMax}\left(10 \cdot t \cdot \mathbf{e}^{\frac{-t}{2}}, t, 0, 24\right)$ $t = 2$

$C(2) = 20e^{-1} \approx 7.36$ mg/L **A1**
Answer a. $20e^{-1}$ mg/L at $t = 2$

b. A second identical dose is taken 6 hours after the first, so for $t \geq 6$ the concentration is $C(t) + C(t-6)$. Find the maximum concentration after the second dose and the time at which it occurs, each correct to two decimal places. 3 marks

Define $g(t) = C(t) + C(t-6)$ for $t \geq 6$. **M1**

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Define $c(t) = 10 \cdot t \cdot \mathbf{e}^{\frac{-t}{2}}$

$\text{fMax}(c(t) + c(t-6), t, 6, 24)$ $t = 7.71544$

$g(7.715\ldots) \approx 8.90$ mg/L **M1 A1**
Answer b. 8.90 mg/L at $t \approx 7.72$
Watch out: the second peak is not 2 hours after the second dose ($t = 8$). The leftover from the first dose is still falling, which pulls the peak earlier. And it is not 2×7.36 either.

Question 5B.2*** (4 marks) Half-life Workbook p. 34

The mass of a radioactive substance is $m(t) = m_0 e^{-kt}$ grams after t days. Its half-life is 8 days. [Notes link: **5.8** Applications: it's just transformations]

a. Show that $k = \frac{1}{8}\log_e 2$. 1 mark

$m_0 e^{-8k} = \frac{1}{2}m_0 \Rightarrow -8k = \log_e \frac{1}{2} = -\log_e 2$ **A1**
Answer a. Shown

b. Find the fraction of the original mass that remains after 24 days. 1 mark

24 days is three half-lives: $\left(\frac{1}{2}\right)^3$ **A1**
Answer b. $\frac{1}{8}$

c. Find the exact time taken for 90% of the substance to decay. 2 marks

10% remains: $e^{-kt} = \frac{1}{10} \Rightarrow kt = \log_e 10$ **M1**
$t = \dfrac{8\log_e 10}{\log_e 2}$ days **A1**
Answer c. $\dfrac{8\log_e 10}{\log_e 2}$ days $(= 8\log_2 10 \approx 26.6)$

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Ways to do it: (1) Solve with k, as above. (2) Rewrite the model as $m = m_0\left(\frac{1}{2}\right)^{t/8}$ and solve $\left(\frac{1}{2}\right)^{t/8} = \frac{1}{10}$. (3) Estimate to check: $\frac{1}{10}$ is between $\frac{1}{8}$ (3 half-lives) and $\frac{1}{16}$ (4 half-lives), so $24 < t < 32$.
Watch out: "90% decays" means 10% is left. Solving $e^{-kt} = 0.9$ answers the wrong question.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 5.1–5.10

Question 5C.1** (1 mark) Tripling time Workbook p. 36

A population is modelled by $P = 200e^{0.1t}$. The time taken for the population to triple is

A. $10\log_e(3)$ **B.** $\frac{1}{10}\log_e(3)$

C. $\log_e(30)$ **D.** $10\log_e(600)$

[Notes link: **5.8** Applications: it's just transformations]
$600 = 200e^{0.1t} \Rightarrow e^{0.1t} = 3 \Rightarrow t = 10\log_e 3$. The starting amount doesn't matter. **A1**
Answer A

Question 5C.2*** (1 mark) Exponential inequality Workbook p. 36

The solution of $e^{-2x} < 4$ is

A. $x < -\log_e(2)$ **B.** $x > -\log_e(2)$

C. $x > \log_e(2)$ **D.** $x < \log_e(2)$

[Notes link: **5.9** Solving non-linear inequalities: sketch it]
$-2x < \log_e 4 = 2\log_e 2$. Dividing by -2 flips the sign: $x > -\log_e 2$. (Sketch $y = e^{-2x}$: it's decreasing, so it's below 4 to the *right*.) **A1**
Answer B

Question 5C.3**** (1 mark) A log with a base less than 1 Workbook p. 36

The solution of $\log_{\frac{1}{2}}(x) > 3$ is

A. $x > \frac{1}{8}$ **B.** $x < \frac{1}{8}$

C. $0 < x < \frac{1}{8}$ **D.** $x > 8$

[Notes link: **5.9** Solving non-linear inequalities: sketch it]
$\log_{\frac{1}{2}}$ is a *decreasing* function, so the inequality flips: $x < \left(\frac{1}{2}\right)^3 = \frac{1}{8}$. The domain also needs $x > 0$.
Another way: rewrite as $-\log_2 x > 3 \Rightarrow \log_2 x < -3 \Rightarrow x < 2^{-3}$.
A1
Answer C

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6 Circular Functions

Section A – Technology free

Notes for this section: Notes 6.1–6.10

Question 6A.1* (3 marks) Exact values Workbook p. 38

Find the exact value of each. [Notes link: **6.3** Quadrants and the reference angle]

a. $\sin\left(\frac{5\pi}{6}\right)$ 1 mark

Quadrant 2, reference angle $\frac{\pi}{6}$, sine positive. **A1**
Answer a. $\frac{1}{2}$
Another way: unit circle: $\frac{5\pi}{6}$ is in quadrant 2 with reference angle $\frac{\pi}{6}$, and sine is positive there.

b. $\cos\left(-\frac{2\pi}{3}\right)$ 1 mark

Quadrant 3, reference angle $\frac{\pi}{3}$, cosine negative. **A1**
Answer b. $-\frac{1}{2}$

c. $\tan\left(\frac{7\pi}{4}\right)$ 1 mark

Quadrant 4, reference angle $\frac{\pi}{4}$, tan negative. **A1**
Answer c. -1
Another way: tan has period π, so $\tan\left(\frac{7\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right) = -1$.

Question 6A.2** (2 marks) Solving a trig equation Workbook p. 38

[Notes link: **6.6** Solving trig equations]

Solve $2\sin(x) = \sqrt{3}$ for $x \in [0, 2\pi]$. 2 marks

$\sin x = \frac{\sqrt{3}}{2}$: reference angle $\frac{\pi}{3}$, sine positive in quadrants 1 and 2. **M1 A1**
Answer $x = \frac{\pi}{3},\ \frac{2\pi}{3}$
Another way: sketch $y = 2\sin(x)$ and $y = \sqrt{3}$ over $[0, 2\pi]$: two intersections, symmetric about $x = \frac{\pi}{2}$. Or CAS solve with the domain.

Question 6A.3**** (5 marks) Quadratics in sine and cosine Workbook p. 39

[Notes link: **6.6** Solving trig equations]

a. Solve $2\sin^2(x) - \sin(x) - 1 = 0$ for $x \in [0, 2\pi]$. 2 marks

$(2\sin x + 1)(\sin x - 1) = 0 \Rightarrow \sin x = 1$ or $\sin x = -\frac{1}{2}$ **M1**
$x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$ **A1**
Answer a. $x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$

b. Solve $2\cos^2(x) + 3\sin(x) - 3 = 0$ for $x \in [0, 2\pi]$. 3 marks

Use $\cos^2 x = 1 - \sin^2 x$ so there's only one trig function: $2 - 2\sin^2 x + 3\sin x - 3 = 0$ **M1**
$2\sin^2 x - 3\sin x + 1 = 0 \Rightarrow (2\sin x - 1)(\sin x - 1) = 0$ **M1**
$\sin x = \frac{1}{2}$ or 1: $x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}$ **A1**
Answer b. $x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}$
Ways to do it: (1) Substitute $u = \sin x$ and solve the quadratic, as above; reject any u outside $[-1, 1]$. (2) If both $\sin^2$ and $\cos$ (or $\cos^2$ and $\sin$) appear, use the Pythagorean identity first. (3) CAS: $\text{solve}(2\cdot(\cos(x))^2 + 3\cdot\sin(x) - 3 = 0, x)\,|\,0 \le x \le 2\cdot\pi$, in radian mode.

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Question 6A.4*** (3 marks) Sine equals a multiple of cosine Workbook p. 39

[Notes link: **6.6** Solving trig equations]

Solve $\sin(2x) = \sqrt{3}\cos(2x)$ for $x \in [0, \pi]$. 3 marks

Divide by $\cos(2x)$: $\tan(2x) = \sqrt{3}$ **M1**
$2x \in [0, 2\pi]$: $2x = \frac{\pi}{3},\ \frac{4\pi}{3}$ **M1**
$x = \frac{\pi}{6},\ \frac{2\pi}{3}$ **A1**
Answer $x = \frac{\pi}{6},\ \frac{2\pi}{3}$
Ways to do it: (1) Divide by cos to get tan, as above. It's safe here because $\cos(2x) = 0$ would force $\sin(2x) = 0$ too, which never happens at the same angle. (2) Graph both sides and find the intersections. (3) CAS solve with the domain.
Watch out: dividing by $\sin$ or $\cos$ is only safe when it can't be zero at a solution. For $\sin x \cos x = \cos x$, factorise instead: $\cos x(\sin x - 1) = 0$.

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Section B – Technology active

Notes for this section: Notes 6.1–6.10

Question 6B.1***** (7 marks) Extended response: hours of daylight Workbook p. 40

The number of hours of daylight in a city on day t of the year is modelled by $D(t) = 12 + 2.4\sin\left(\dfrac{2\pi(t-80)}{365}\right)$, for $1 \le t \le 365$. [Notes link: **6.8** Sketching circular functions]

a. State the greatest and least number of hours of daylight. 1 mark

12 ± 2.4 **A1**
Answer a. 14.4 hours and 9.6 hours

b. Find the number of days in the year with more than 13.5 hours of daylight. 3 marks

Solve $D(t) = 13.5$: **M1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(12 + 2.4 \cdot \sin\left(\dfrac{2 \cdot \pi \cdot (t-80)}{365}\right) = 13.5, t\right) \mid 1 \le t \le 365 \qquad t = 119.219 \text{ or } t = 223.281$

$D > 13.5$ for $119.22 < t < 223.28$, i.e. days 120 to 223 inclusive. **M1**
$223 - 120 + 1 = 104$ days **A1**
Answer b. 104 days
Watch out: t is a whole number of days. Count the whole days strictly inside the interval; $223.28 - 119.22 \approx 104.06$ happens to round to the same thing, but that isn't the method.

c. A second city is modelled by $E(t) = 12 + a\sin\left(\dfrac{2\pi(t-80)}{365}\right)$, $a > 0$. Each year, the interval of time during which it has more than 13.5 hours of daylight is exactly 150 days long. Find a, correct to two decimal places. 3 marks

$E > 13.5$ when $\sin\theta > \frac{1.5}{a}$, where $\theta = \frac{2\pi(t-80)}{365}$. By symmetry about the peak the interval is $\theta_1 < \theta < \pi - \theta_1$, of width $\pi - 2\theta_1$. **M1**
Width in θ for 150 days: $\frac{2\pi \times 150}{365}$, so $\theta_1 = \frac{1}{2}\left(\pi - \frac{300\pi}{365}\right) \approx 0.2797$ **M1**
$\frac{1.5}{a} = \sin(\theta_1) \Rightarrow a \approx 5.43$ **A1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(a \cdot \sin\left(\dfrac{1}{2} \cdot \left(\pi - \dfrac{300 \cdot \pi}{365}\right)\right) = 1.5, a\right) \qquad a = 5.43287$

Answer c. $a \approx 5.43$
Ways to do it: (1) Symmetry about the maximum, as above. (2) The interval is centred on the peak (day 171.25), so it starts 75 days earlier at $t = 96.25$: solve $E(96.25) = 13.5$ for a. (3) CAS slider on a: solve $E(t) = 13.5$ and adjust a until the two solutions are 150 apart.

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Question 6B.2**** (4 marks) Time above a height, forwards and backwards Workbook p. 41

A carriage on a Ferris wheel is $h(t) = 12 - 10\cos\left(\dfrac{\pi t}{15}\right)$ metres above the ground t seconds after it starts. [Notes link: **6.9** Trig inequalities]

a. For how many seconds in each rotation is the carriage more than 17 m above the ground? 2 marks

$12 - 10\cos\left(\frac{\pi t}{15}\right) > 17 \Rightarrow \cos\left(\frac{\pi t}{15}\right) < -\frac{1}{2}$ **M1**
In one rotation $\frac{\pi t}{15} \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$, i.e. $10 < t < 20$: 10 seconds. **A1**
Answer a. 10 seconds

b. A second wheel has $H(t) = 12 - 10\cos(nt)$. Its carriage is more than 17 m above the ground for exactly 8 seconds in each rotation. Find n. 2 marks

Above 17 m when the angle nt is between $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$: a third of each rotation, whatever the speed. **M1**
So the period is $3 \times 8 = 24$ seconds: $\frac{2\pi}{n} = 24 \Rightarrow n = \frac{\pi}{12}$ **A1**
Answer b. $n = \frac{\pi}{12}$
Ways to do it: (1) Fraction of a cycle: angle width $\frac{2\pi}{3}$ out of 2π is $\frac{1}{3}$, as above. (2) Solve directly: $t_2 - t_1 = \frac{4\pi}{3n} - \frac{2\pi}{3n} = \frac{2\pi}{3n} = 8$. (3) CAS: solve $H(t) = 17$ in terms of n, subtract the first two solutions and set equal to 8, or use a slider on n.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 6.1–6.10

Question 6C.1*** (1 mark) General solution Workbook p. 42

The general solution of $\tan(2x) = 1$ is

A. $x = \frac{\pi}{4} + n\pi,\ n \in Z$ **B.** $x = \frac{\pi}{8} + n\pi,\ n \in Z$

C. $x = \frac{\pi}{8} + \frac{n\pi}{2},\ n \in Z$ **D.** $x = \frac{\pi}{8} + 2n\pi,\ n \in Z$

[Notes link: **6.7** General solutions]
$2x = \frac{\pi}{4} + n\pi$ (tan repeats every π), then divide *everything* by 2. **A1**
Answer C

Question 6C.2** (1 mark) Pythagorean identity Workbook p. 42

If $\cos(x) = -\frac{5}{13}$ and $\pi < x < \frac{3\pi}{2}$, then $\sin(x)$ is equal to

A. $\frac{12}{13}$ **B.** $-\frac{12}{13}$

C. $\frac{8}{13}$ **D.** $-\frac{8}{13}$

[Notes link: **6.10** The Pythagorean identity]
$\sin^2 x = 1 - \frac{25}{169} = \frac{144}{169}$, and sine is negative in quadrant 3. **A1**
Answer B

Question 6C.3**** (1 mark) Smallest value for one solution Workbook p. 42

The equation $\cos\left(ax + \frac{\pi}{6}\right) = 1$, where $a > 0$, has exactly one solution for $x \in [0, 2]$ when

A. $a < \frac{11\pi}{12}$ **B.** $\frac{11\pi}{12} \leq a < \frac{23\pi}{12}$

C. $a \geq \frac{23\pi}{12}$ **D.** $\frac{11\pi}{6} \leq a < \frac{23\pi}{6}$

[Notes link: **6.7** General solutions]
$ax + \frac{\pi}{6} = 2k\pi$: the positive solutions are $x = \frac{11\pi}{6a},\ \frac{23\pi}{6a}, \ldots$ One solution when $\frac{11\pi}{6a} \leq 2$ and $\frac{23\pi}{6a} > 2$. **A1**
Answer B

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7 Composite and Inverse Functions

Section A – Technology free

Notes for this section: Notes 7.1–7.6

Question 7A.1* (3 marks) Composite rules Workbook p. 44

Let $f(x) = x^2 + 1$ and $g(x) = 2x - 3$. [Notes link: **7.1** Composite functions and the rule]

a. Find $f\big(g(x)\big)$. 1 mark

Replace the x in f with $2x-3$. **A1**
Answer a. $(2x-3)^2+1$

b. Find $g\big(f(x)\big)$. 1 mark

$2(x^2+1)-3$ **A1**
Answer b. $2x^2-1$

c. Evaluate $f\big(g(2)\big)$. 1 mark

$g(2)=1$, $f(1)=2$ **A1**
Answer c. 2
Another way: work from the inside out with numbers: find $g(2)$ first, then put that value into f.

Question 7A.2** (3 marks) Inverse of an exponential Workbook p. 44

Let $f(x) = 3e^{2x} - 1$. [Notes link: **7.4** Inverse functions]

Find the rule and domain of f^{-1}. 3 marks

Swap: $x = 3e^{2y} - 1 \Rightarrow e^{2y} = \frac{x+1}{3}$ **M1**
$y = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$ **M1**
$\operatorname{ran} f = (-1,\infty)$, so that's the domain of f^{-1}. **A1**
Answer $f^{-1} : (-1,\infty) \to R,\ f^{-1}(x) = \frac{1}{2}\log_e\left(\frac{x+1}{3}\right)$
Another way: CAS: $\text{solve}(x = 3\cdot \mathbf{e}^{2\cdot y} - 1, y)$.

Question 7A.3*** (4 marks) Inverse by completing the square Workbook p. 45

Let $f : (-\infty, a] \to R,\ f(x) = x^2 + 2x - 3$. [Notes link: **7.4** Inverse functions]

a. Find the largest value of a for which f has an inverse function. 1 mark

$f(x) = (x+1)^2 - 4$: the turning point is at $x = -1$, and f is one-to-one to the left of it. **A1**
Answer a. $a = -1$

b. For this value of a, find the rule and domain of f^{-1}. 3 marks

Swap: $x = (y+1)^2 - 4 \Rightarrow y + 1 = \pm\sqrt{x+4}$ **M1**
$\operatorname{ran} f^{-1} = \operatorname{dom} f = (-\infty, -1]$, so take the **minus**: $f^{-1}(x) = -1 - \sqrt{x+4}$ **A1**
$\operatorname{dom} f^{-1} = \operatorname{ran} f = [-4, \infty)$ **A1**
Answer b. $f^{-1}(x) = -1 - \sqrt{x+4}$, domain $[-4,\infty)$
Ways to do it: (1) Complete the square, then swap, as above. (2) Swap first, then use the quadratic formula on $y^2 + 2y - (3+x) = 0$: $y = -1 \pm \sqrt{4+x}$. (3) CAS: $\text{solve}(x = y^2 + 2\cdot y - 3, y)$ gives both branches; *you* choose using the domain of f. (4) Check with a point: $f(-3) = 0$, and $f^{-1}(0) = -1 - 2 = -3$. ✓
Watch out: taking the $+$ out of habit. The domain of f is the *left* half of the parabola, so the inverse is the lower branch.

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Question 7A.4**** (4 marks) Restricting so that a composite exists Workbook p. 45

Let $f(x) = \sqrt{x-1}$ with its maximal domain, and $g : R \to R,\ g(x) = 4 - x^2$. [Notes link: **7.2** Finding the domain and range of a composite]

a. Explain why $f \circ g$ is not defined. 1 mark

$\operatorname{ran} g = (-\infty, 4]$ is not a subset of $\operatorname{dom} f = [1, \infty)$. **A1**
Answer a. $\operatorname{ran} g \not\subseteq \operatorname{dom} f$

b. Find the largest domain D for g so that $f \circ g$ is defined. 2 marks

Need $g(x) \geq 1$: $4 - x^2 \geq 1 \Rightarrow x^2 \leq 3$ **M1**
$D = [-\sqrt{3}, \sqrt{3}]$ **A1**
Answer b. $D = [-\sqrt{3}, \sqrt{3}]$

c. With this domain, state the rule and range of $f \circ g$. 1 mark

$f\big(g(x)\big) = \sqrt{3 - x^2}$; on D, $3 - x^2 \in [0, 3]$. **A1**
Answer c. $\sqrt{3 - x^2}$, range $[0, \sqrt{3}]$
Ways to do it: (1) Solve "output of g lands in $\operatorname{dom} f$", as above. (2) Sketch g and keep only the part of the graph with $y \geq 1$. (3) Find the rule first and take its maximal domain: this works here, but it can give the wrong answer when g itself has a restricted domain, so method (1) is safer.

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Section B – Technology active

Notes for this section: Notes 7.1–7.6

Question 7B.1***** (8 marks) An exponential, its inverse and a circle Workbook p. 47

Let $f(x) = e^{ax} - b$, where $a, b > 0$, over its maximal domain. [Notes link: **7.6** Inverses in calculus: tangency, gradients and areas]

a. Find the rule and domain of the inverse function f^{-1}. 2 marks

$x = e^{ay} - b \Rightarrow y = \dfrac{1}{a}\log_e(x+b)$ **A1**

$\text{dom}\, f^{-1} = \text{ran}\, f = (-b, \infty)$ **A1**

Answer a. $f^{-1} : (-b,\infty) \to R,\ f^{-1}(x) = \frac{1}{a}\log_e(x+b)$

b. Find b in terms of a such that the graphs of $y = f(x)$ and $y = f^{-1}(x)$ are tangent to each other. 2 marks

The trick: f is strictly increasing, so it can only touch its inverse on the line $y = x$. Use $f'(x) = 1$ and $f(x) = x$, and leave the inverse out of it.

$f'(x) = ae^{ax} = 1 \Rightarrow e^{ax} = \dfrac{1}{a}$, so $x = -\dfrac{\log_e(a)}{a}$. **M1**

$f(x) = x$: $\dfrac{1}{a} - b = -\dfrac{\log_e(a)}{a}$, so $b = \dfrac{1+\log_e(a)}{a}$ **A1**

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Define $f(x) = \mathbf{e}^{a \cdot x} - b$

solve $\left(f(x) = x \text{ and } \dfrac{d}{dx}(f(x)) = 1, b, x\right) \mid a > 0$ $\qquad b = \dfrac{\ln(a)+1}{a}$ and $x = \dfrac{-\ln(a)}{a}$

Answer b. $b = \dfrac{1+\log_e(a)}{a}$ (which needs $a > \frac{1}{e}$ so that $b > 0$)

Watch out: type $a \cdot x$ with the multiplication sign, and put the restriction $a > 0$ at the *end* of the solve.

c. For $a = 1$ and a certain value of b, the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect at $x = 1$. Find the area of the region bounded by the graphs, correct to four decimal places. 3 marks

The intersections are on $y = x$, so $f(1) = 1$: $e - b = 1$, $b = e - 1$. **M1**

Other intersection: $e^x - (e-1) = x \Rightarrow x = -1.49375\ldots$ **M1**

By symmetry in $y = x$, the area is twice the area between $y = x$ and f:

$A = 2\displaystyle\int_{-1.49375\ldots}^{1} \left(x - \left(e^x - e + 1\right)\right) dx \approx 2.3511$ **A1**

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solve$(\mathbf{e}^x - (e-1) = x, x)$ $\qquad x = -1.49375$ or $x = 1.$

$2 \cdot \cdot \displaystyle\int_{-1.4937535}^{1} \cdot (x - (\mathbf{e}^x - e + 1)) \cdot dx$ $\qquad 2.35114$

Answer c. ≈ 2.3511 square units

Watch out: keep all the decimals of $-1.49375\ldots$ in the terminal (store it, or copy it down from the line above). Rounding it to -1.49 changes the fourth decimal place of the area.

d. Let C be the circle that circumscribes the region described in part c. Find the area of C, correct to two decimal places. 1 mark

The two corners of the region are $(1,1)$ and $(-1.49375, -1.49375)$, and the region is thin enough to fit inside the circle that has those two points as the ends of a diameter. Diameter $= \sqrt{2}(1+1.49375)$, so the area is $\pi\left(\dfrac{\sqrt{2} \times 2.49375}{2}\right)^2 \approx 9.77$. **A1**

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Answer d. ≈ 9.77 square units
Another way: check it really fits. The widest part of the region is where $f'(x) = 1$, at $x = 0$: the gap from $y = x$ to f there is $\frac{e-2}{\sqrt{2}} \approx 0.51$, far less than the radius 1.76.

Question 7B.2*** (7 marks) Composites from a table Workbook p. 48

The functions f and g both have domain $\{1,2,3,4,5\}$. Their rules are not given, but all of their values are in the table. [Notes link: **7.1** Composite functions and the rule]

x	1	2	3	4	5
$f(x)$	3	5	2	3	1
$g(x)$	4	1	5	2	3

a. Evaluate (i) $f\big(g(2)\big)$ (ii) $g\big(f(f(1))\big)$. 2 marks

The trick: work from the inside out, one table lookup at a time.
(i) $g(2) = 1$, then $f(1) = 3$ **A1**
(ii) $f(1) = 3$, then $f(3) = 2$, then $g(2) = 1$ **A1**
Answer a. (i) 3 (ii) 1

b. Find all values of x for which $f\big(g(x)\big) = 3$. 2 marks

Now work from the outside in. $f(u) = 3$ when $u = 1$ or $u = 4$, so I need $g(x) = 1$ or $g(x) = 4$. **M1**
$g(x) = 1 \Rightarrow x = 2$ and $g(x) = 4 \Rightarrow x = 1$ **A1**
Answer b. $x = 1$ or $x = 2$
Watch out: f hits 3 twice, so there are two inside values to chase. Stopping at $u = 1$ loses a solution.

c. The function g is one-to-one, so it has an inverse. Find $g^{-1}\big(f(5)\big)$. 1 mark

$f(5) = 1$, and $g^{-1}(1)$ is the x with $g(x) = 1$, which is $x = 2$. **A1**
Answer c. 2

d. Explain why f does not have an inverse function, and state the number of solutions of $g\big(f(x)\big) = 2$. 2 marks

$f(1) = f(4) = 3$: two inputs give the same output, so f is not one-to-one. **A1**
$g(u) = 2$ only when $u = 4$, and f never outputs 4 (its range is $\{1,2,3,5\}$), so there are no solutions. **A1**
Answer d. f is many-to-one ($f(1) = f(4)$); no solutions

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Section C – Multiple choice (technology active)

Notes for this section: Notes 7.1–7.6

Question 7C.1**** (1 mark) Intersections off the line $y = x$ Workbook p. 49

The number of points of intersection of the graphs of $f(x) = -x^3$ and its inverse is

A. 0 **B.** 1

C. 2 **D.** 3

[Notes link: **7.5** Intersections of f and f^{-1}]
f is *decreasing*, so intersections don't have to be on $y = x$. $f^{-1}(x) = -x^{\frac{1}{3}}$; $-x^3 = -x^{\frac{1}{3}} \Rightarrow x^9 = x \Rightarrow x = 0, \pm 1$. The points are $(0,0)$, $(1,-1)$ and $(-1,1)$. Solving $f(x) = x$ would only find $(0,0)$. **A1**
Answer D

Question 7C.2*** (1 mark) Gradient of an inverse Workbook p. 49

A one-to-one function f has $f(2) = 5$ and $f'(2) = 4$. The gradient of the graph of $y = f^{-1}(x)$ at $x = 5$ is

A. 4 **B.** -4

C. $\frac{1}{4}$ **D.** $\frac{1}{5}$

[Notes link: **7.6** Inverses in calculus: tangency, gradients and areas]
The point $(2,5)$ on f matches $(5,2)$ on f^{-1}, and the gradients are reciprocals. **A1**
Answer C

Question 7C.3** (1 mark) ♢ Power functions Workbook p. 49

Which one of the following has maximal domain R?

A. $y = x^{\frac{1}{2}}$ **B.** $y = x^{-1}$

C. $y = x^{\frac{1}{3}}$ **D.** $y = x^{-\frac{1}{2}}$

[Notes link: **7.7** Power functions (reference)]
Cube roots work for negatives; square roots don't, and negative powers exclude 0. **A1**
Answer C

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8 Curve Sketching

Section A – Technology free

Notes for this section: Notes 8.1–8.4, and 13.7 in the CAS Guide

Question 8A.1** (4 marks) Finding asymptotes Workbook p. 51

State the equations of all asymptotes of each graph. [Notes link: **8.2** Asymptotes, properly]

a. $y = \dfrac{3}{x^2-4}+1$ 2 marks

Vertical: denominator 0 at $x=\pm 2$. **M1**
Horizontal: as $x \to \pm\infty$, $\frac{3}{x^2-4} \to 0$, so $y \to 1$. **A1**
Answer a. $x=2$, $x=-2$, $y=1$

b. $y = \dfrac{x^2+1}{x}$ 2 marks

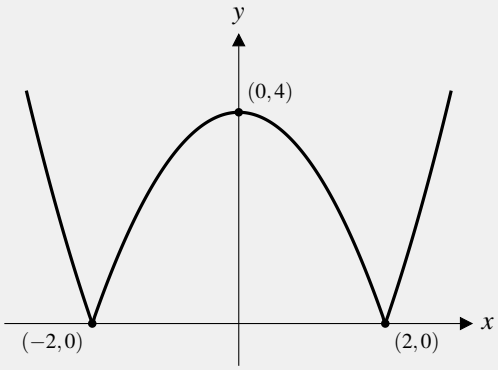
Split: $y = x + \dfrac{1}{x}$. **M1**
Vertical $x=0$; as $x \to \pm\infty$, $\frac{1}{x} \to 0$, so the graph approaches the oblique line $y=x$. **A1**
Answer b. $x=0$ and $y=x$

Question 8A.2** (3 marks) ♢ Absolute value Workbook p. 51

[Notes link: **8.3** Other techniques]

Sketch $y=|x^2-4|$, labelling all axis intercepts and any points where the graph is not smooth. 3 marks

Sketch $y=x^2-4$, then reflect the part below the x-axis up above it. **M1**
Intercepts $(\pm 2,0)$ (sharp corners there) and $(0,4)$ (now a local maximum). **A1 A1**



Answer W shape: corners at $(\pm 2,0)$, local max $(0,4)$
Another way: write it as a hybrid function: x^2-4 for $x \le -2$ or $x \ge 2$, and $4-x^2$ for $-2 < x < 2$.

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Question 8A.3*** (3 marks) Reciprocal graphs Workbook p. 52

Let $f(x) = x^2 - 1$. [Notes link: **8.3** Other techniques]

Without sketching it accurately, describe the key features of the graph of $y = \dfrac{1}{f(x)}$: its asymptotes, where it is positive and negative, and its turning point. 3 marks

Zeros of f at $x = \pm 1$ become vertical asymptotes; $f \to \infty$ gives $\frac{1}{f} \to 0$, so $y = 0$ is a horizontal asymptote. **M1**
$f > 0$ for $|x| > 1$ and $f < 0$ for $|x| < 1$; $\frac{1}{f}$ has the same sign. **M1**
f's minimum $(0, -1)$ becomes a maximum of $\frac{1}{f}$ at $(0, -1)$. **A1**
Answer Asymptotes $x = \pm 1$, $y = 0$; positive for $|x| > 1$, negative between; local max $(0, -1)$

Question 8A.4**** (3 marks) Finding a rule from its asymptotes Workbook p. 52

[Notes link: **8.2** Asymptotes, properly]

The graph of $y = \dfrac{ax+b}{x+c}$ has asymptotes $x = 2$ and $y = 3$, and passes through $(0, 1)$. Find a, b and c. 3 marks

Vertical asymptote where the denominator is zero: $c = -2$. **A1**
As $x \to \pm\infty$, $y \to a$: $a = 3$. **A1**
$(0, 1)$: $\frac{b}{-2} = 1 \Rightarrow b = -2$. **A1**
Answer $a = 3$, $b = -2$, $c = -2$
Another way: write it as $y = 3 + \frac{k}{x-2}$ and use the point: $3 - \frac{k}{2} = 1 \Rightarrow k = 4$, so $y = 3 + \frac{4}{x-2} = \frac{3x-2}{x-2}$.

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Section B – Technology active

Notes for this section: Notes 8.1–8.4, and 13.7 in the CAS Guide

Question 8B.1*** (5 marks) Sketching with technology Workbook p. 53

Let $f(x) = x^2e^{-x}$. [Notes link: **13.7** CAS Guide: sketching on the CAS]

a. Find the coordinates of the stationary points, giving exact values. 2 marks

$f'(x) = e^{-x}(2x - x^2) = xe^{-x}(2-x) = 0$ **M1**
$x = 0$: $(0,0)$; $x = 2$: $(2, 4e^{-2})$ **A1**

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$\text{solve}\left(\frac{d}{dx}\left(x^2 \cdot \mathbf{e}^{-x}\right) = 0, x\right)$ $x = 0$ or $x = 2$

Answer a. $(0,0)$ and $(2, 4e^{-2})$

b. Find the x-coordinates of the points of inflection, giving exact values. 2 marks

$f''(x) = e^{-x}(x^2 - 4x + 2) = 0$ **M1**

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$\text{solve}\left(\frac{d^2}{dx^2}\left(x^2 \cdot \mathbf{e}^{-x}\right) = 0, x\right)$ $x = 2 - \sqrt{2}$ or $x = 2 + \sqrt{2}$

A1

Answer b. $x = 2 \pm \sqrt{2}$

c. State the equation of the asymptote. 1 mark

As $x \to \infty$, e^{-x} shrinks faster than x^2 grows, so $f(x) \to 0$. **A1**
Answer c. $y = 0$ (as $x \to \infty$)

Question 8B.2***** (7 marks) A full analysis with technology Workbook p. 53

Let $f(x) = \dfrac{\log_e(x)}{x}$. [Notes link: **13.7** CAS Guide: sketching on the CAS]

a. State the maximal domain of f, its x-intercept, and the equations of its asymptotes. 2 marks

Domain $(0, \infty)$; x-intercept $(1, 0)$. **A1**
As $x \to 0^+$, $f(x) \to -\infty$: asymptote $x = 0$. As $x \to \infty$, $f(x) \to 0$: asymptote $y = 0$. **A1**
Answer a. $(0, \infty)$; $(1, 0)$; asymptotes $x = 0$ and $y = 0$

b. Find the exact coordinates of the turning point and the point of inflection. 2 marks

TI-Nspire | Calculator page, menu 3 1 solve

Define $f(x) = \dfrac{\ln(x)}{x}$

$\text{solve}\left(\frac{d}{dx}\left(f(x)\right) = 0, x\right)$ $x = \mathbf{e}$

$\text{solve}\left(\frac{d^2}{dx^2}\left(f(x)\right) = 0, x\right)$ $x = \mathbf{e}^{\frac{3}{2}}$

Maximum $\left(e, \frac{1}{e}\right)$ **A1**

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Point of inflection $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$ **A1**

Answer b. Maximum $\left(e, \frac{1}{e}\right)$; inflection $\left(e^{\frac{3}{2}}, \frac{3}{2}e^{-\frac{3}{2}}\right)$

Another way: by hand: $f'(x) = \frac{1-\log_e x}{x^2}$ and $f''(x) = \frac{2\log_e x - 3}{x^3}$.

c. Hence find the values of k for which the equation $\log_e(x) = kx$ has (i) two solutions, (ii) exactly one solution.

3 marks

For $x > 0$, $\log_e(x) = kx \Leftrightarrow f(x) = k$: count where the horizontal line $y = k$ cuts the graph of f. **M1**

(i) Two cuts when $0 < k < \frac{1}{e}$. **A1**

(ii) One cut when $k = \frac{1}{e}$ (at the maximum) or $k \leq 0$ (only the left-hand part, since $f > 0$ for all $x > 1$). **A1**

Answer c. (i) $0 < k < \frac{1}{e}$ (ii) $k = \frac{1}{e}$ or $k \leq 0$

Ways to do it: (1) Rearrange so the parameter is alone ($f(x) = k$) and use the graph of f, as above. (2) Tangency: the line $y = kx$ touches $y = \log_e x$ when $\log_e x = kx$ and $\frac{1}{x} = k$, giving $x = e$, $k = \frac{1}{e}$; then compare slopes. (3) CAS slider on k with $y = \log_e x$ and $y = kx$.

Watch out: $k \leq 0$ is easy to miss: a line through the origin with zero or negative gradient still cuts the log graph once.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 8.1–8.4, and 13.7 in the CAS Guide

Question 8C.1** (1 mark) Addition of ordinates Workbook p. 55

Let $h(x) = f(x) + g(x)$. If $f(2) = 3$ and $g(2) = -3$, then the graph of h must

A. have a turning point at $x = 2$

B. have an asymptote at $x = 2$

C. have an x-intercept at $x = 2$

D. pass through $(2, 3)$

[Notes link: **8.3** Other techniques]
Add the y-values: $h(2) = 3 + (-3) = 0$. **A1**
Answer C

Question 8C.2*** (1 mark) Reciprocal graphs Workbook p. 55

The graph of $y = f(x)$ has x-intercepts at $x = -1$ and $x = 2$ only. The graph of $y = \dfrac{1}{f(x)}$ has

A. x-intercepts at $x = -1$ and $x = 2$

B. vertical asymptotes at $x = -1$ and $x = 2$

C. x-intercepts at $x = -1$ and $x = \frac{1}{2}$

D. turning points at $x = -1$ and $x = 2$

[Notes link: **8.3** Other techniques]
Where $f(x) = 0$, $\frac{1}{f(x)}$ is undefined: zeros become vertical asymptotes. **A1**
Answer B

Question 8C.3*** (1 mark) Piecing the clues together Workbook p. 55

The graph of $y = x\log_e(x)$ has a local minimum at

A. $(1, 0)$

B. $(e,\ e)$

C. $\left(\frac{1}{e},\ -\frac{1}{e}\right)$

D. $\left(\frac{1}{e},\ -1\right)$

[Notes link: **8.4** Worked example: piecing the clues together]
$\frac{dy}{dx} = \log_e x + 1 = 0 \Rightarrow x = e^{-1}$, and $y = e^{-1} \times (-1)$. **A1**
Answer C

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9 Differentiation

Section A – Technology free

Notes for this section: Notes 9.1–9.12

Question 9A.1* (5 marks) The toolkit Workbook p. 57

Differentiate each with respect to x. [Notes link: **9.2** The differentiation toolkit]

a. $3x^4 - \dfrac{2}{x} + \sqrt{x}$ 1 mark

Write as $3x^4 - 2x^{-1} + x^{\frac{1}{2}}$ first. **A1**
Answer a. $12x^3 + \dfrac{2}{x^2} + \dfrac{1}{2\sqrt{x}}$

b. $(2x+1)^5$ 1 mark

Chain rule: $5(2x+1)^4 \times 2$ **A1**
Answer b. $10(2x+1)^4$

c. $x^2 \log_e x$ 1 mark

Product rule: $2x\log_e x + x^2 \cdot \frac{1}{x}$ **A1**
Answer c. $2x\log_e x + x$

d. $\dfrac{\cos x}{x}$ 2 marks

Quotient rule: $\dfrac{x(-\sin x) - \cos x \cdot 1}{x^2}$ **M1 A1**
Answer d. $-\dfrac{x\sin x + \cos x}{x^2}$
Watch out: the numerator is $gf' - fg'$ (bottom times derivative of top first). Swapping the order flips the sign.
Another way: product rule on $x^{-1}\cos x$. Check any derivative on the CAS with the $\frac{d}{dx}$ template.

Question 9A.2** (3 marks) Average and instantaneous rates of change Workbook p. 58

The height of a plant is $h(t) = 2\sqrt{t}$ cm after t weeks. [Notes link: **9.1** What is a derivative?]

a. Find the average rate of change of the height over the first 9 weeks. 1 mark

$\dfrac{h(9)-h(0)}{9-0} = \dfrac{6}{9}$ **A1**
Answer a. $\frac{2}{3}$ cm/week

b. Find the instantaneous rate of change of the height when $t = 9$. 2 marks

$h'(t) = \dfrac{1}{\sqrt{t}}$ **M1**
$h'(9) = \frac{1}{3}$ **A1**
Answer b. $\frac{1}{3}$ cm/week

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Question 9A.3*** (5 marks) Motion in a straight line Workbook p. 58

A particle moves in a straight line so that its position is $x(t) = t^3 - 6t^2 + 9t$ metres at time t seconds, $t \geq 0$. [Notes link: **9.1** What is a derivative?]

a. Find the times at which the particle is at rest, and its acceleration at each of those times. 2 marks

$v(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3) = 0 \Rightarrow t = 1,\ 3$ **A1**
$a(t) = 6t - 12$: $a(1) = -6$ and $a(3) = 6$ m/s^2 **A1**
Answer a. $t = 1$ ($a = -6$) and $t = 3$ ($a = 6$)

b. State the values of t for which the particle is moving in the negative direction. 1 mark

$v < 0$ **A1**
Answer b. $1 < t < 3$

c. Find the total distance travelled in the first 4 seconds. 2 marks

Positions: $x(0) = 0$, $x(1) = 4$, $x(3) = 0$, $x(4) = 4$. **M1**
Distance $= 4 + 4 + 4 = 12$ m **A1**
Answer c. 12 m
Watch out: the displacement is only $x(4) - x(0) = 4$ m. Distance needs each change of direction handled separately.

Question 9A.4**** (5 marks) Optimisation: a closed cylinder Workbook p. 59

A closed cylindrical can must hold 16π cm^3. [Notes link: **9.9** Absolute maximums and minimums (optimisation)]

a. Show that the surface area is $A = 2\pi r^2 + \dfrac{32\pi}{r}$, where r cm is the radius. 2 marks

$\pi r^2 h = 16\pi \Rightarrow h = \frac{16}{r^2}$ **M1**
$A = 2\pi r^2 + 2\pi r h = 2\pi r^2 + \frac{32\pi}{r}$ **A1**
Answer a. Shown

b. Find the radius and height that minimise the surface area, and the minimum surface area. 3 marks

$\frac{dA}{dr} = 4\pi r - \frac{32\pi}{r^2} = 0 \Rightarrow r^3 = 8 \Rightarrow r = 2$ **M1**
$\frac{d^2A}{dr^2} = 4\pi + \frac{64\pi}{r^3} > 0$, so it's a minimum. **M1**
$h = 4$, $A = 8\pi + 16\pi = 24\pi$ cm^2 **A1**
Answer b. $r = 2$ cm, $h = 4$ cm, $A = 24\pi$ cm^2
Ways to do it: Every optimisation question is the same four steps: (1) write the quantity in terms of *one* variable using the constraint; (2) state the domain; (3) differentiate and solve $= 0$; (4) justify max or min (second derivative, sign of the derivative, or compare with the endpoints). On the CAS use fMin, but still write the function and the equation you solved.

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Section B – Technology active

Notes for this section: Notes 9.1–9.12

Question 9B.1*** (3 marks) Newton's method Workbook p. 60

[Notes link: **9.12** Approximating x-intercepts]

Use Newton's method with $x_0 = 2$ to find x_2 for $f(x) = x^3 - 2x - 5$. Give x_2 correct to four decimal places. 3 marks

$x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)},\ f'(x) = 3x^2 - 2.$ **M1**

$x_1 = 2 - \frac{-1}{10} = 2.1$ **M1**

$x_2 = 2.1 - \frac{0.061}{11.23} \approx 2.0946$ **A1**

TI-Nspire | Calculator page

$x - \dfrac{x^3 - 2\cdot x - 5}{3\cdot x^2 - 2} \mid x = 2$	2.1
$x - \dfrac{x^3 - 2\cdot x - 5}{3\cdot x^2 - 2} \mid x = 2.1$	2.09457

Answer $x_2 \approx 2.0946$

Question 9B.2***** (2 marks) No negative roots Workbook p. 60

[Notes link: **9.11** Families of functions (slider questions)]

Let $f(x) = x^2 + k\sin(x)$ over its maximal domain, where $k \in R$. Find the minimum value of k, correct to two decimal places, for which all roots of $f(x) = 0$ are non-negative. 2 marks

The trick: $x = 0$ is always a root, so the question is really "when is there no root to the left of 0?". Use a slider for k and watch the left-hand side of the graph.

$k > 0$ fails: just left of 0 the graph is below the axis, and $f(-\pi) = \pi^2 > 0$, so there is a negative root. $k = 0$ works.
For $k < 0$ there are no roots in $(-\pi, 0)$, because $k\sin(x) > 0$ there. As k decreases, the first negative root appears when the graph touches the axis between -2π and $-\pi$: $f(x) = 0$ and $f'(x) = 0$. **M1**
$k = -20.1748\ldots$ (at $x \approx -4.275$). So all roots are non-negative for $-20.1748\ldots < k \leq 0$, and to two decimal places the minimum is -20.17. **A1**

TI-Nspire | Calculator page

Define $f(x) = x^2 + k\cdot\sin(x)$	
$\text{solve}\left(f(x) = 0 \text{ and } \dfrac{d}{dx}(f(x)) = 0, k, x\right) \mid -2\cdot\pi < x < -\pi$	$k = -20.1749$ and $x = -4.27478$

Answer $k = -20.17$ (all roots are non-negative for $-20.1748\ldots < k \leq 0$)
Watch out: domain at the end of the solve again. And $k = -20.17$ is inside the allowed interval, but -20.18 is not: check which way the rounding has to go.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 9.1–9.12

Question 9C.1** (1 mark) Tangent Workbook p. 61

The tangent to the graph of $y = \sqrt{x}$ at $x = 4$ has equation

A. $y = \frac{1}{4}x + 1$ **B.** $y = \frac{1}{4}x + 2$

C. $y = \frac{1}{2}x$ **D.** $y = 4x - 14$

[Notes link: **9.8** Tangents and normals]
$\frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{4}$ at $x = 4$, through $(4,2)$: $y - 2 = \frac{1}{4}(x-4)$. **A1**
Answer A

Question 9C.2*** (1 mark) Absolute maximum Workbook p. 61

The absolute maximum value of $f : [-2,3] \to R,\ f(x) = x^3 - 3x$ is

A. 2 **B.** -2

C. 3 **D.** 18

[Notes link: **9.9** Absolute maximums and minimums (optimisation)]
The local max is $f(-1) = 2$, but the endpoint $f(3) = 18$ is higher. Always check the endpoints.
Watch out: the question asks for the value (y), not where it occurs (x). **A1**
Answer D

Question 9C.3**** (1 mark) Tangential graphs Workbook p. 61

The graphs of $y = e^x$ and $y = x + k$ touch when k equals

A. 0 **B.** 1

C. e **D.** -1

[Notes link: **9.10** Tangential graphs]
Same gradient: $e^x = 1 \Rightarrow x = 0$. Same y-value there: $e^0 = 0 + k$. **A1**
Answer B

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10 Integration

Section A – Technology free

Notes for this section: Notes 10.1–10.10

Question 10A.1* (3 marks) Antiderivatives Workbook p. 63

Find an antiderivative of each. [Notes link: **10.2** Integration formulas]

a. $6x^2 - \dfrac{4}{x^2}$ 1 mark

$6x^2 - 4x^{-2} \to 2x^3 + 4x^{-1}$ **A1**
Answer a. $2x^3 + \dfrac{4}{x}$

b. e^{2x+1} 1 mark

Linear inside: divide by 2. **A1**
Answer b. $\frac{1}{2}e^{2x+1}$

c. $\dfrac{1}{3x-2}$ 1 mark

A1
Answer c. $\frac{1}{3}\log_e|3x-2|$
Another way: check any antiderivative by differentiating it: $\frac{d}{dx}\left(\frac{1}{3}\log_e(3x-2)\right) = \frac{1}{3x-2}$.

Question 10A.2** (2 marks) Trapezium rule Workbook p. 63

[Notes link: **10.5** Estimating areas]

Use the trapezium rule with two strips of equal width to estimate $\displaystyle\int_1^3 \frac{1}{x}\,dx$. 2 marks

Width 1; heights $f(1) = 1$, $f(2) = \frac{1}{2}$, $f(3) = \frac{1}{3}$. **M1**
$\frac{1}{2}\left[1 + 2\left(\frac{1}{2}\right) + \frac{1}{3}\right] = \frac{1}{2} \times \frac{7}{3}$ **A1**
Answer $\frac{7}{6}$
Another way: check against the exact value $\log_e 3 \approx 1.0986$: the estimate should be slightly larger, because $y = \frac{1}{x}$ is concave up.

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Question 10A.3*** (3 marks) An unknown limit Workbook p. 64

[Notes link: **10.3** The fundamental theorem of calculus]

a. Find the value of $a > 1$ for which $\int_1^a (2x-3)\,dx = 6$. 2 marks

$\left[x^2-3x\right]_1^a = a^2-3a+2 = 6$ **M1**
$a^2-3a-4=0 \Rightarrow a=4$ (as $a>1$) **A1**
Answer a. $a=4$

b. Find the exact value of k for which $\int_0^k e^{2x}\,dx = 4$. 1 mark

$\frac{1}{2}\left(e^{2k}-1\right)=4 \Rightarrow e^{2k}=9 \Rightarrow k=\log_e 3$ **A1**
Answer b. $k=\log_e 3$

Question 10A.4**** (3 marks) A function defined by an integral Workbook p. 64

Let $F(x)=\int_0^x \left(t^2-4t+3\right)dt$. [Notes link: **10.3** The fundamental theorem of calculus]

Find $F'(x)$, and hence find the coordinates of the local maximum of the graph of F. 3 marks

By the fundamental theorem, $F'(x)=x^2-4x+3=(x-1)(x-3)$ **A1**
F' changes from positive to negative at $x=1$: local maximum. **M1**
$F(1)=\left[\frac{t^3}{3}-2t^2+3t\right]_0^1=\frac{4}{3}$ **A1**
Answer $F'(x)=x^2-4x+3$; local maximum $\left(1,\frac{4}{3}\right)$
Another way: integrate first: $F(x)=\frac{x^3}{3}-2x^2+3x$, then differentiate. The theorem just saves the work.

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Section B – Technology active

Notes for this section: Notes 10.1–10.10

Question 10B.1***** (6 marks) Extended response: flow in and flow out Workbook p. 65

Water flows into a reservoir at a rate of $I(t) = 20 + 10\sin\left(\frac{\pi t}{12}\right)$ ML per hour and is released at a constant 24 ML per hour, for $0 \le t \le 24$. The reservoir holds 500 ML when $t = 0$. [Notes link: **10.3** The fundamental theorem of calculus]

a. Find the times at which the volume of water stops decreasing and starts increasing, and stops increasing and starts decreasing, correct to two decimal places. 2 marks

The volume is stationary when the net rate $I(t) - 24$ is zero. **M1**

TI-Nspire | Calculator page, menu 3 1 solve

$\text{solve}\left(20 + 10 \cdot \sin\left(\frac{\pi \cdot t}{12}\right) = 24, t\right) \mid 0 \le t \le 24$ $\quad t = 1.57188$ or $t = 10.4281$

Starts increasing at $t \approx 1.57$; starts decreasing at $t \approx 10.43$. **A1**

Answer a. $t \approx 1.57$ and $t \approx 10.43$

b. Find the maximum volume of water in the reservoir, correct to the nearest megalitre. 2 marks

$V(10.4281) = 500 + \int_0^{10.4281} \big(I(t) - 24\big)\,dt$ **M1**

≈ 531 ML **A1**

Answer b. 531 ML

c. Find the minimum volume of water in the reservoir during the 24 hours. 2 marks

Candidates: the local minimum at $t \approx 1.57$ and the endpoint $t = 24$. **M1**

$V(1.57188) \approx 496.9$; $V(24) = 500 + \int_0^{24} \big(I(t) - 24\big)\,dt = 500 - 96 = 404$ ML. Minimum 404 ML, at $t = 24$. **A1**

Answer c. 404 ML (at $t = 24$)

Ways to do it: The volume is $V(t) = V(0) + \int_0^t (\text{rate in} - \text{rate out})\,dt$. (1) Find when the net rate is zero, then evaluate V at those times *and at the endpoints*. (2) Define $v(t)$ on the CAS with the integral and use fMax/fMin on $[0, 24]$. (3) Graph $v(t)$ and read off the highest and lowest points.

Watch out: the absolute minimum is at the endpoint, not at the stationary point. Always check both.

Question 10B.2*** (3 marks) Trapezium rule with technology Workbook p. 66

[Notes link: **10.5** Estimating areas]

Use the trapezium rule with four strips of equal width to estimate $\int_0^2 e^{-x^2}\,dx$, correct to four decimal places. Find the percentage error in this estimate, correct to two decimal places. 3 marks

Width 0.5: $\frac{0.5}{2}\big(f(0) + 2f(0.5) + 2f(1) + 2f(1.5) + f(2)\big)$ **M1**

TI-Nspire | Calculator page

Define $f(x) = \mathbf{e}^{-x^2}$

$0.25 \cdot (f(0) + 2 \cdot (f(0.5) + f(1) + f(1.5)) + f(2))$ $\quad 0.880619$

$\int_0^2 \cdot f(x) \cdot dx$ $\quad 0.882081$

Estimate ≈ 0.8806 **A1**

Error $\frac{0.882081 - 0.880619}{0.882081} \times 100 \approx 0.17\%$ **A1**

Answer 0.8806; error $\approx 0.17\%$

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Section C – Multiple choice (technology active)

Notes for this section: Notes 10.1–10.10

Question 10C.1** (1 mark) Area between curves Workbook p. 67

The area enclosed by the graphs of $y = x^2$ and $y = 2x$ is

A. $\frac{2}{3}$ **B.** $\frac{4}{3}$

C. $\frac{8}{3}$ **D.** 4

[Notes link: **10.6** Exact areas]
They meet at $x = 0, 2$, with the line on top: $\int_0^2 \left(2x - x^2\right) dx = 4 - \frac{8}{3}$. **A1**
Answer B

Question 10C.2*** (1 mark) Splitting an integral Workbook p. 67

If $\int_1^5 f(x)\,dx = 7$ and $\int_3^5 f(x)\,dx = 2$, then $\int_1^3 \big(f(x) + 2\big)\,dx$ is equal to

A. 5 **B.** 7

C. 9 **D.** 11

[Notes link: **10.7** Manipulating integrals]
$\int_1^3 f = 7 - 2 = 5$, and $\int_1^3 2\,dx = 4$ (height 2, width 2). **A1**
Answer C

Question 10C.3**** (1 mark) Differentiating an integral Workbook p. 67

$\frac{d}{dx}\left(\int_1^x \sqrt{1+t^3}\,dt\right)$ is equal to

A. $\sqrt{1+x^3}$ **B.** $\sqrt{1+x^3} - \sqrt{2}$

C. $\dfrac{3x^2}{2\sqrt{1+x^3}}$ **D.** $\sqrt{1+t^3}$

[Notes link: **10.3** The fundamental theorem of calculus]
The fundamental theorem: differentiating undoes the integral and replaces t with x. The lower limit 1 only adds a constant, which differentiates to 0. **A1**
Answer A

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11 Probability and Statistics

Section A – Technology free

Notes for this section: Notes 11.1–11.10

Question 11A.1** (3 marks) Counting Workbook p. 69

[Notes link: **11.2** Counting methods]

a. How many different arrangements are there of the letters of LEVEL? 1 mark

5 letters, two L's and two E's: $\frac{5!}{2!2!}$ **A1**
Answer a. 30

b. A committee of 3 is chosen from 5 boys and 4 girls. How many committees have at least one girl? 2 marks

Complement: all committees minus all-boy committees. **M1**
$\binom{9}{3} - \binom{5}{3} = 84 - 10$ **A1**
Answer b. 74
Another way: count by cases: 1 girl $4\binom{5}{2} = 40$, 2 girls $\binom{4}{2} \times 5 = 30$, 3 girls $\binom{4}{3} = 4$; total 74.

Question 11A.2** (2 marks) The empirical rule Workbook p. 69

X is normally distributed with mean 50 and standard deviation 4. Use the 68–95–99.7% rule to estimate: [Notes link: **11.8** The normal distribution]

a. $\Pr(X > 58)$ 1 mark

58 is 2 sd above the mean: $\frac{1-0.95}{2}$ **A1**
Answer a. 0.025
Another way: draw the bell curve and mark $\mu \pm \sigma$, $\mu \pm 2\sigma$: 58 is two standard deviations above the mean.

b. $\Pr(46 < X < 58)$ 1 mark

-1 sd to $+2$ sd: $0.34 + 0.475$ **A1**
Answer b. 0.815

Question 11A.3*** (3 marks) A fair game Workbook p. 70

In a game, a fair die is rolled once. The player wins \$12 for a six, \$3 for a four or a five, and nothing otherwise. [Notes link: **11.5** Discrete random variables]

a. It costs \$5 to play. Find the player's expected profit per game. 2 marks

Expected winnings: $12 \times \frac{1}{6} + 3 \times \frac{2}{6} = 3$ **M1**
Expected profit $= 3 - 5 = -\$2$ (a loss of \$2 per game on average) **A1**
Answer a. $-\$2$

b. The prize for a six is changed to \$$w$ so that the game is fair at \$5 to play. Find w. 1 mark

Fair means expected profit zero: $\frac{w}{6} + 1 = 5$ **A1**
Answer b. $w = 24$

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Question 11A.4**** (6 marks) A pdf: mode and median Workbook p. 70

The continuous random variable X has probability density function $f(x) = kx(4-x^2)$ for $0 \leq x \leq 2$, and $f(x) = 0$ elsewhere. [Notes link: **11.7** Continuous random variables]

a. Show that $k = \frac{1}{4}$. 1 mark

$k\int_0^2 (4x - x^3)\,dx = k\left[2x^2 - \frac{x^4}{4}\right]_0^2 = 4k = 1$ **A1**

Answer a. Shown

b. Find the mode of X. 2 marks

The mode is where f is greatest: $f'(x) = \frac{1}{4}(4-3x^2) = 0$ **M1**

$x = \frac{2}{\sqrt{3}}$ (f is zero at both ends, so this is the maximum) **A1**

Answer b. $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

c. Find the exact median of X. 3 marks

$\int_0^m \frac{1}{4}(4x - x^3)\,dx = \frac{1}{2} \Rightarrow \frac{1}{4}\left(2m^2 - \frac{m^4}{4}\right) = \frac{1}{2}$ **M1**

$m^4 - 8m^2 + 8 = 0 \Rightarrow m^2 = 4 \pm 2\sqrt{2}$ **M1**

m must be in $[0,2]$, so $m^2 = 4 - 2\sqrt{2}$: $m = \sqrt{4-2\sqrt{2}}$ (≈ 1.08) **A1**

Answer c. $m = \sqrt{4-2\sqrt{2}}$

Ways to do it: Mean, median and mode are three different calculations: mean $= \int x f(x)\,dx$; median solves $\int_{\text{start}}^m f(x)\,dx = \frac{1}{2}$; mode maximises f. On the CAS, solve $\left(\int_0^m \cdot f(x) \cdot dx = \frac{1}{2}, m\right) | 0 < m < 2$.

Watch out: the quadratic in m^2 gives two values. Reject $4+2\sqrt{2}$ because that m is outside $[0,2]$.

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Section B – Technology active

Notes for this section: Notes 11.1–11.10

Question 11B.1*** (3 marks) What a confidence level means Workbook p. 72

Forty different researchers each take their own random sample from the same population and each calculates a 95% confidence interval for the same proportion p. [Notes link: **11.9** Sampling]

a. How many of the 40 intervals would you expect *not* to contain p? 1 mark

5% of 40 **A1**
Answer a. 2

b. Find the probability that all 40 intervals contain p, correct to four decimal places. 2 marks

Each contains p with probability 0.95, independently: 0.95^{40} **M1**
≈ 0.1285 **A1**
Answer b. 0.1285
Watch out: "95% confident" describes the method over many samples. It does not mean there is a 95% chance that p is in one particular interval you've already calculated.

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Question 11B.2***** (12 marks) Extended response: a random launch speed Workbook p. 73

A machine fires tennis balls from ground level at the origin. The path of a ball is modelled by

$$y = x - \frac{10x^2}{v^2}, \qquad x \geq 0,$$

where x metres is the horizontal distance from the machine, y metres is the height of the ball and v metres per second is the launch speed. [Notes link: **11.8** The normal distribution]

a. Find, in terms of v, how far from the machine the ball lands. 1 mark

The trick: turn the condition on the path into an interval for the random variable. The probability is then one line.

$y = 0 \Rightarrow x\left(1 - \frac{10x}{v^2}\right) = 0$ **A1**

Answer a. $\frac{v^2}{10}$ metres

b. Find the maximum height of the ball, in terms of v. 2 marks

By symmetry the maximum is halfway, at $x = \frac{v^2}{20}$ **M1**

$y = \frac{v^2}{20} - \frac{10}{v^2} \times \frac{v^4}{400} = \frac{v^2}{40}$ **A1**

Answer b. $\frac{v^2}{40}$ metres

Another way: $\frac{dy}{dx} = 1 - \frac{20x}{v^2} = 0$.

A net 2 metres high stands 30 metres from the machine. A shot is *in* if the ball passes over the net and lands no more than 45 metres from the machine.

c. Find the exact interval of values of v for which a shot is in. 3 marks

Over the net: height at $x = 30$ is more than 2: $30 - \frac{9000}{v^2} > 2 \Rightarrow v^2 > \frac{2250}{7}$ **M1**

Lands in: $\frac{v^2}{10} \leq 45 \Rightarrow v^2 \leq 450$ **M1**

$\sqrt{\frac{2250}{7}} < v \leq \sqrt{450}$, i.e. about $17.93 < v \leq 21.21$ **A1**

Answer c. $\frac{15\sqrt{70}}{7} < v \leq 15\sqrt{2}$

d. The launch speed is a normally distributed random variable V with a mean of 20 and a standard deviation of 2.

Find the probability that a shot is in, correct to four decimal places. 2 marks

All the graph work was to turn "the shot is in" into an interval for V. Now it is one normal probability: **M1**

TI-Nspire | Calculator page, menu 5 5 2 Normal Cdf

$\text{normCdf}(\sqrt{2250/7}, \sqrt{450}, 20, 2)$	0.577791

A1

Answer d. 0.5778

Watch out: keep the exact bounds in the CAS. With 17.93 and 21.21 you get 0.5771, which is wrong in the third decimal place.

e. Ten balls are fired, independently. Find the probability that at least eight of the shots are in, correct to four decimal places. 2 marks

$X \sim \text{Bi}(10,\ 0.57779\ldots)$ **M1**

$\Pr(X \geq 8) = \text{binomCdf}(10, 0.577791, 8, 10) = 0.1341$ **A1**

Answer e. 0.1341

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f. The mean launch speed can be changed, while the standard deviation stays at 2. Find the mean, correct to two decimal places, that makes the probability of a shot being in as large as possible. 2 marks

A normal curve is symmetric with its peak at the mean, so a fixed interval captures the most area when the mean is at the **middle of the interval**. **M1**

$\mu = \frac{1}{2}\left(\sqrt{\frac{2250}{7}} + \sqrt{450}\right) = 19.57$ **A1**

Answer f. $\mu = 19.57$ (the probability rises to 0.5885)

Ways to do it: (1) Symmetry, as above. (2) CAS: $\text{fMax}\left(\text{normCdf}\left(\sqrt{2250/7}, \sqrt{450}, m, 2\right), m\right)$.
(3) Graph $f1(x) = \text{normCdf}\left(\sqrt{2250/7}, \sqrt{450}, x, 2\right)$ and use menu 6 (Analyse Graph) for the maximum.

Watch out: these questions look like two topics, but the probability part is tiny. The marks are in turning the condition on the path into an interval for the random variable, with the correct $<$ or $\leq$ at each end.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 11.1–11.10

Question 11C.1**** (1 mark) Conditional probability with a pdf Workbook p. 75

X has pdf $f(x) = 2x$ for $0 \leq x \leq 1$ (and 0 elsewhere). $\Pr(X > 0.5 \mid X > 0.25)$ is equal to

A. 0.75 **B.** 0.8

C. 0.5 **D.** 0.9375

[Notes link: **11.7** Continuous random variables]
$\Pr(X > a) = \int_a^1 2x\,dx = 1 - a^2$. So $\frac{\Pr(X>0.5)}{\Pr(X>0.25)} = \frac{0.75}{0.9375}$. **A1**
Answer B

Question 11C.2*** (1 mark) A small population Workbook p. 75

A box holds 5 red and 5 blue pens. Two are chosen at random without replacement, and $\hat{P}$ is the proportion of red pens in the sample. $\Pr\left(\hat{P} = \frac{1}{2}\right)$ is equal to

A. $\frac{1}{2}$ **B.** $\frac{5}{9}$

C. $\frac{2}{9}$ **D.** $\frac{1}{4}$

[Notes link: **11.9** Sampling]
Small population, so not binomial. $\hat{P} = \frac{1}{2}$ means exactly 1 red: $\frac{\binom{5}{1}\binom{5}{1}}{\binom{10}{2}} = \frac{25}{45}$. **A1**
Answer B

Question 11C.3** (1 mark) Expected value of a function Workbook p. 75

X takes the values 1, 2 and 3 with probabilities 0.2, 0.5 and 0.3. $E\left(X^2\right)$ is equal to

A. 4.41 **B.** 4.9

C. 2.1 **D.** 14

[Notes link: **11.10** Expected value of a function of X]
Replace x with x^2: $1(0.2) + 4(0.5) + 9(0.3)$.
Watch out: 4.41 is $\left(E(X)\right)^2$. Not the same thing.
A1
Answer B

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12 Miscellaneous

Section A – Technology free

Notes for this section: Notes 12.1–12.4, and the CAS Guide (13.1–13.9)

Question 12A.1** (2 marks) Tracing an algorithm Workbook p. 77

[Notes link: **12.1** Pseudocode]

What value does this algorithm print?

```
s ← 0
for i from 1 to 4
    s ← s + i^2
end for
print s
```

2 marks

Trace: $s = 1, 5, 14, 30$. **M1 A1**

Answer 30

Question 12A.2*** (3 marks) Bisection Workbook p. 77

[Notes link: **12.1** Pseudocode]

The bisection method is applied to $f(x) = x^2 - 3$ starting with the interval $[1, 2]$. Find the interval after three iterations.

```
a ← 1, b ← 2
for i from 1 to 3
    m ← (a + b)/2
    if f(a)*f(m) < 0 then
        b ← m
    else
        a ← m
    end if
end for
```

3 marks

i	a	b	m	$f(m)$	action
1	1	2	1.5	-0.75	same sign as $f(a) = -2$: $a \leftarrow 1.5$
2	1.5	2	1.75	0.0625	opposite sign: $b \leftarrow 1.75$
3	1.5	1.75	1.625	-0.359	same sign: $a \leftarrow 1.625$

M1 M1 A1

Answer $[1.625,\ 1.75]$

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Question 12A.3*** (1 mark) Functional equations Workbook p. 78

[Notes link: **12.2** Functional equations]

Which one of the following functions satisfies $f(x+y)=f(x)f(y)$ for all real x and y?

A. $f(x)=3x$ **B.** $f(x)=x^3$

C. $f(x)=e^{3x}$ **D.** $f(x)=3e^x$

1 mark

$e^{3(x+y)}=e^{3x}e^{3y}$. (D fails: $3e^{x+y}\neq 9e^xe^y$.) **A1**
Answer C

Question 12A.4**** (2 marks) A functional equation proof Workbook p. 78

[Notes link: **12.2** Functional equations]

Let $f(x)=kx+c$, where $k,c\in R$. Show that $f(x+y)=f(x)+f(y)$ for all x,y only if $c=0$. 2 marks

$f(x+y)=kx+ky+c$, and $f(x)+f(y)=kx+ky+2c$. **M1**
Equal for all $x,y\Rightarrow c=2c\Rightarrow c=0$. **A1**
Answer Shown

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Section B – Technology active

Notes for this section: Notes 12.1–12.4, and the CAS Guide (13.1–13.9)

Question 12B.1**** (3 marks) A simulation Workbook p. 79

[Notes link: **12.1** Pseudocode]

```
count ← 0
for i from 1 to 10000
  d1 ← random integer from 1 to 6
  d2 ← random integer from 1 to 6
  if d1 + d2 ≥ 10 then count ← count + 1
end for
print count/10000
```

a. State what the algorithm estimates, and the value you would expect it to print, correct to three decimal places. 2 marks

The probability that the sum of two fair dice is at least 10. **A1**
Outcomes $(4,6),(5,5),(6,4),(5,6),(6,5),(6,6)$: $\frac{6}{36} \approx 0.167$ **A1**
Answer a. $\Pr(\text{sum} \geq 10)$; about 0.167

b. Explain why running the algorithm twice is likely to print two different values. 1 mark

The rolls are random, so the count varies from run to run; with 10 000 trials the results will be close to 0.167 but not identical. **A1**
Answer b. Random variation between runs
Another way: on the CAS, $\text{randInt}(1,6,10000)$ in a Lists & Spreadsheet page generates all the rolls at once.

Question 12B.2*** (6 marks) Round only at the end Workbook p. 79

A population is modelled by $P(t) = 500e^{kt}$, where t is in years, and $P(3) = 800$. [Notes link: **12.3** Exam conventions]

a. Find the exact value of k, and find $P(20)$ correct to the nearest whole number. 2 marks

$e^{3k} = 1.6 \Rightarrow k = \frac{1}{3}\log_e(1.6)$ **A1**
$P(20) = 500e^{\frac{20}{3}\log_e(1.6)} = 500 \times 1.6^{\frac{20}{3}} \approx 11\,475$ **A1**
Answer a. $k = \frac{1}{3}\log_e(1.6)$; $P(20) \approx 11\,475$

b. A student rounds k to 0.16 first. Find the value they get for $P(20)$, and the size of their error. 2 marks

$500e^{0.16\times 20} = 500e^{3.2} \approx 12\,266$ **A1**
$12\,266 - 11\,475 = 791$ too many. **A1**
Answer b. $\approx 12\,266$, which is about 791 too high
Watch out: rounding k changed it by only 0.003, but the error is multiplied by 20 and then goes through an exponential. Store the exact value ($k := \frac{\ln(1.6)}{3}$) and round on the last line only.

c. Find the time at which the population reaches 2000. Give your answer in years, correct to two decimal places. 2 marks

$500e^{kt} = 2000 \Rightarrow t = \dfrac{\log_e(4)}{k} = \dfrac{3\log_e(4)}{\log_e(1.6)}$ **M1**
≈ 8.85 years **A1**
Answer c. 8.85 years
Watch out: with $k = 0.16$ this comes out as 8.66. Also give the units: the question has them.

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Section C – Multiple choice (technology active)

Notes for this section: Notes 12.1–12.4, and the CAS Guide (13.1–13.9)

Question 12C.1*** (1 mark) Completing an algorithm Workbook p. 81

The algorithm below should carry out Newton's method. The missing line is

```
x ← 1
for i from 1 to 5
    ______________
end for
print x
```

A. `x ← f(x)/df(x)` **B.** `x ← x − df(x)/f(x)`

C. `x ← x − f(x)/df(x)` **D.** `x ← x + f(x)/df(x)`

[Notes link: **12.1** Pseudocode]
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$: f on top, f' on the bottom, and the new value replaces the old x. **A1**
Answer C

Question 12C.2** (1 mark) Exact or decimal? Workbook p. 81

A question says: "Find the x-intercept of the graph of $y = e^x - 2$." Which answer gets the mark?

A. 0.69 **B.** 0.693

C. $\log_e(2)$ **D.** $\log(2)$

[Notes link: **12.3** Exam conventions]
No accuracy was asked for, so the answer must be exact, and the base of the log has to be written. **A1**
Answer C

Question 12C.3**** (1 mark) A long algorithm: Newton's method with a stopping rule Workbook p. 81

Consider the algorithm below, written in pseudocode.

```
define f(x)
    return x^3 − 2x − 5
define df(x)
    return 3x^2 − 2
x ← 2
n ← 0
while |f(x)| > 0.01
    x ← x − f(x)/df(x)
    n ← n + 1
end while
print n, x
```

The values printed by the algorithm, with `x` correct to four decimal places, are

A. `1, 2.1000` **B.** `2, 2.0946`

C. `3, 2.0946` **D.** `2, 2.1000`

[Notes link: **12.1** Pseudocode]
Trace it in a table. Start: $x = 2$, $f(2) = -1$, and $|-1| > 0.01$, so the loop runs. Pass 1: $x = 2 - \frac{-1}{10} = 2.1$, $n = 1$; $f(2.1) = 0.061 > 0.01$, so it runs again. Pass 2: $x = 2.1 - \frac{0.061}{11.23} = 2.0946$, $n = 2$; $f(2.0946) \approx 0.0002$, which is not more than 0.01, so the loop stops.
Watch out: the condition is checked *before* each pass, using the current x. A stops one pass early; C does one pass too

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many.

A1

Answer B

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