Free Practice Workbook

2027 Edition

Written and taught by **Leonard Shen**

TOPICS

1 Logic and Proof
2 Circular Functions
3 Vectors, Lines and Planes
4 Complex Numbers
5 Differential Calculus and Sketching
6 Integration Techniques
7 Applications of Integration
8 Differential Equations
9 Kinematics
10 Vector Calculus
11 Probability and Statistics

USING THIS WORKBOOK

A free sample of the CoastEd **Question Workbook**: a spread of questions from every topic, from the basics to a separator, copied unchanged from the full book. The full workbook has 489 questions, and with the **Notes** and **Worked Solutions** it comes with every CoastEd course (coasted.org). Notes references (for example "Notes 9.4") point to sections of the CoastEd Notes.

The topics match the **Notes**, in the same order. Every topic has **Section A** (technology free: shorter questions done by hand, like Examination 1), **Section B** (technology active: the questions that need the CAS and the long extended-response questions, like Section B of Examination 2) and **Section C** (multiple choice, like Section A of Examination 2), then a **Short answers** page. Later topics build on earlier ones, so expect to use old skills in new places.

Tick **Done**, **Marked** and **Redo** at the top of each question as you go. Anything marked Redo goes in your mistake log.

Worked solutions are in the separate **Free Practice Worked Solutions**, with the other methods that work listed for each question. You may use the VCAA formula sheet. **Read "How to use this workbook" before you start.**

⋆ and ⋆⋆	The basics: these should be automatic
⋆⋆⋆	Standard exam questions
⋆⋆⋆⋆ and ⋆⋆⋆⋆⋆	Hard questions and separators

This workbook contains **99** questions.

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How to use this workbook

Every question has a level, shown by the stars after its number. The levels tell you what to expect of yourself. Work through a topic in order the first time. After that, use the stars: the one- to three-star questions until they are automatic, then the four- and five-star questions until none of them surprise you.

Level	What it is	What you should be able to do
⋆ and ⋆⋆	The basics	Do them **straight away, with no mistakes, like clockwork**. No looking at the notes, no long pause to remember the method.
⋆⋆⋆	Standard exam questions	Get them right with care. A slip now and then is normal; the same slip twice goes in your mistake log.
⋆⋆⋆⋆ and ⋆⋆⋆⋆⋆	Hard questions and separators	Expect to get stuck. These are the question types that decide the top marks.

If the easy ones weren't clockwork

If a one- or two-star question took you more than about a minute per mark, if you had to look something up, or if you made *any* mistake, you are rusty on that skill and one go at it here is not enough practice. The hard questions are built out of the easy ones, so fix it before moving on:

- **Textbook:** each topic in the Notes ends with a *Textbook questions* list. Do every second question of the matching exercise until they feel automatic.
- **Ask an AI for more:** type in the question and ask for ten more like it, the same difficulty, with worked answers at the end. Do them on paper before you look. Then redo the workbook question, and tick **Redo** until you get it right first time.

The hard questions are where the marks are decided

Give each four- or five-star question a proper attempt (about two minutes per mark) before you open the solutions, because working out *how to start* is the skill being tested. Most rely on one idea that is easy to miss, and that idea will come back in a different disguise in your exam. Study the worked solution, then **redo the question from a blank page a week later**.

What is in each topic

- **Section A** is technology free: shorter questions done by hand, like Examination 1. **Section B** is technology active: the questions that need the CAS and the long extended-response questions, like Section B of Examination 2. **Section C** is multiple choice, like Section A of Examination 2. Each section starts easy and gets harder. Question 4A.9 is Topic 4, Section A, Question 9.
- These questions are a sample from the full CoastEd Question Workbook, which covers every style of question in the course, from one-mark basics to the last part of an extended-response question.
- The workbook shows only the question. Its title, and the notes section it tests, are in the Worked Solutions, so that nothing on the page tells you which method to use. There is a **Short answers** page at the end of each topic for a quick check.
- A question marked ◇ **non-essential** goes past what the exam usually asks, or is a style that rarely appears. Leave it until the rest of the topic is solid.

How to read the solutions

The **Free Practice Worked Solutions** follow this workbook with the same question numbers, and each solution shows the workbook page of its question. A solution can have up to four parts:

- the **full working**, set out the way you should set it out for full marks, with the marks at the right: **M1** is a method mark and **A1** is an answer mark;
- **The trick** (on the harder questions): the one idea that unlocks the question;
- **Watch out**: a detail that was put there to catch you (a domain, a sign, a rounding instruction, a solution that has to be rejected), for your mistake log if it caught you;
- **Ways to do it**: every technique that works for that question, by hand and on the TI-Nspire. Read these even when you got the question right, then redo the question with one of the other methods. Knowing two ways is how you check your own answers in an exam.

On the CAS, start a new problem for every question in Sections B and C (doc, 4 Insert, 1 Problem), so an old $f(x)$ or stored value can't quietly change the next answer (see the CAS Guide at the back of the Notes).

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1 Logic and Proof

Section A – Technology free

You may use the formula sheet.

Question 1A.1* (7 marks) ☐ Done ☐ Marked ☐ Redo

a. Write down the converse of "if n is a multiple of 4, then n is even", and state whether the converse is true, giving a reason. 2 marks

b. Write down the contrapositive of "if $x^2 > 9$, then $x > 3$ or $x < -3$". 1 mark

c. Write down the negation of "for all $n \in N$, $n^2 + n$ is even". Which of the statement and its negation is true? 2 marks

d. Show that the statement "for all $x \in R$, if $x^2 = 4$ then $x = 2$" is false. 1 mark

e. Write the statement "every real number has a real cube root" using the quantifiers $\forall$ and $\exists$. 1 mark

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Question 1A.2** (9 marks) ☐ Done ☐ Marked ☐ Redo

a. Prove by induction that $1+3+5+\cdots+(2n-1)=n^2$ for all $n \in N$. 3 marks

b. Prove by induction that 5^n-1 is divisible by 4 for all $n \in N$. 3 marks

c. Prove by induction that $2^n > n$ for all $n \in N$. 3 marks

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Question 1A.3*** (7 marks) ☐ Done ☐ Marked ☐ Redo

a. Prove that $a^2 + b^2 \geq 2ab$ for all $a, b \in R$. 1 mark

b. Prove that $x + \dfrac{4}{x} \geq 4$ for all $x > 0$. 2 marks

c. Express $\dfrac{1}{(2k-1)(2k+1)}$ in partial fractions. 2 marks

d. Hence show that $\displaystyle\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$. 2 marks

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Question 1A.4**** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Let $x > -1$. Prove by induction that $(1+x)^n \geq 1+nx$ for all $n \in N$. 3 marks

b. Prove by induction that $2^n \geq n^2$ for all integers $n \geq 4$. 3 marks

c. Prove by induction that $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2} \leq 2-\frac{1}{n}$ for all $n \in N$. 4 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 1B.1*** (7 marks) ☐ Done ☐ Marked ☐ Redo

A sequence is defined by $a_1 = 1$ and $a_{n+1} = a_n + 2n + 1$ for $n \in N$.

a. Use your calculator to find a_2, a_3, a_4, a_5. 1 mark

b. Conjecture a formula for a_n in terms of n. 1 mark

c. Prove your conjecture by mathematical induction. 3 marks

d. Find the smallest n such that $a_1 + a_2 + \cdots + a_n > 1000$. 2 marks

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Question 1B.2**** (8 marks) ☐ Done ☐ Marked ☐ Redo

a. Evaluate $1 \cdot 1! + 2 \cdot 2! + 3 \cdot 3! + 4 \cdot 4!$. 1 mark

b. Conjecture a formula for $\sum_{k=1}^{n} k \cdot k!$. 1 mark

c. Prove your formula by induction. 3 marks

d. Prove by induction that $n! > 2^n$ for all integers $n \geq 4$. 3 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 1C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

A proof by contradiction that $\sqrt{5}$ is irrational should begin by assuming that

A. $\sqrt{5} = \dfrac{p}{q}$, where $p, q \in Z$, $q \neq 0$ and $\dfrac{p}{q}$ is in lowest terms

B. $\sqrt{5}$ is irrational

C. $\sqrt{5} = \dfrac{p}{q}$, where p and q are both odd

D. 5 is not a perfect square

Answer: (A) (B) (C) (D)

Question 1C.2** (1 mark) ☐ Done ☐ Marked ☐ Redo

The statement $2^n > n^2$ is to be proved by induction for all integers $n \geq n_0$. The smallest valid base case n_0 is

A. 1 **B.** 3 **C.** 5 **D.** 4

Answer: (A) (B) (C) (D)

Question 1C.3** (1 mark) ☐ Done ☐ Marked ☐ Redo

In proving $1+3+5+\cdots+(2n-1) = n^2$ by induction, after assuming the result for $n = k$ you must show that

A. $1+3+\cdots+(2k-1) = k^2$ **B.** $1+3+\cdots+(2k+1) = k^2+1$

C. $1+3+\cdots+(2k-1)+(2k+1) = (k+1)^2$ **D.** $2k+1 = (k+1)^2 - k^2$ for $k = 1$

Answer: (A) (B) (C) (D)

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Logic and Proof – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

1A.1 [1.1] **a.** $n = 2$ is even but not a multiple of 4 **b.** "If $-3 \leq x \leq 3$, then $x^2 \leq 9$" **c.** see Solutions **d.** shown in the Solutions **e.** $\forall x \in R,\ \exists y \in R$ such that $y^3 = x$

1A.2 [1.6] **a.** shown in the Solutions **b.** shown in the Solutions **c.** shown in the Solutions

1A.3 [1.7] **a.** shown in the Solutions **b.** shown in the Solutions **c.** see Solutions **d.** shown in the Solutions

1A.4 [1.6] **a.** see Solutions **b.** shown in the Solutions **c.** shown in the Solutions

1B.1 [1.6] **a.** $a_2 = 4, a_3 = 9, a_4 = 16, a_5 = 25$ **b.** $a_n = n^2$ **c.** shown in the Solutions **d.** $n = 13$ gives 819 and $n = 14$ gives 1015, so $n = 14$

1B.2 [1.6] **a.** $1 + 4 + 18 + 96 = 119$ **b.** $1 = 2! - 1, 5 = 3! - 1, 23 = 4! - 1, 119 = 5! - 1$, so $\sum_{k=1}^{n} k \cdot k! = (n+1)! - 1$ **c.** shown in the Solutions **d.** shown in the Solutions

Section C (multiple choice): 1 A 2 C 3 C

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2 Circular Functions

Section A – Technology free

You may use the formula sheet.

Question 2A.1* (6 marks) ☐ Done ☐ Marked ☐ Redo

Find the exact value of each of the following.

a. $\sec\left(\frac{5\pi}{6}\right)$ 1 mark

b. $\cot\left(-\frac{\pi}{4}\right)$ 1 mark

c. $\operatorname{cosec}\left(\frac{7\pi}{6}\right)$ 1 mark

d. $\sin^{-1}\left(-\frac{1}{2}\right)$ 1 mark

e. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ 1 mark

f. $\tan\left(\cos^{-1}\left(\frac{1}{3}\right)\right)$ 1 mark

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Question 2A.2** (10 marks) ☐ Done ☐ Marked ☐ Redo

Solve each equation for x over the given domain.

a. $2\sin^2(x) - \sin(x) - 1 = 0, \quad x \in [0, 2\pi]$ 3 marks

b. $\sec^2(x) = 4, \quad x \in [0, \pi]$ 2 marks

c. $\sin(2x) = \cos(x), \quad x \in [0, \pi]$ 2 marks

d. $2\cos^2(x) + 3\sin(x) = 3, \quad x \in [0, 2\pi]$ 3 marks

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Question 2A.3*** (8 marks) ☐ Done ☐ Marked ☐ Redo

Prove each of the following identities.

a. $\operatorname{cosec}(2x) - \cot(2x) = \tan(x)$ 2 marks

b. $\cos^4(x) - \sin^4(x) = \cos(2x)$ 2 marks

c. $\sin(3x) = 3\sin(x) - 4\sin^3(x)$ 2 marks

d. $\sqrt{1-\sin(2a)} = \cos(a) - \sin(a)$, for $0 \leq a \leq \dfrac{\pi}{4}$ 2 marks

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Question 2A.4**** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Show that $\sin\left(2\cos^{-1}(x)\right) = 2x\sqrt{1-x^2}$ for $x \in [-1,1]$. 2 marks

b. Express $\cos\left(\tan^{-1}(x)\right)$ and $\sin\left(2\tan^{-1}(x)\right)$ in terms of x, without any circular functions. 3 marks

c. Find the maximal domain and the range of $f(x) = \sin^{-1}\left(x^2-1\right)$. 2 marks

d. Solve $2\sin^{-1}(x) = \cos^{-1}(x)$. 3 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 2B.1*** (8 marks) ☐ Done ☐ Marked ☐ Redo

Let $f: R \to R$, $f(x) = 2\tan^{-1}(x-1) + \pi$.

a. State the range of f and the equations of its asymptotes. 2 marks

b. Find the rule and domain of f^{-1}. 3 marks

c. Explain why every point of intersection of the graphs of f and f^{-1} lies on the line $y = x$. 1 mark

d. Find the coordinates of the point of intersection of the graphs of f and f^{-1}, correct to three decimal places. 2 marks

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Question 2B.2***** (11 marks) ☐ Done ☐ Marked ☐ Redo

Let $f\colon [-1,1] \to R$, $f(x) = \sin^{-1}(x) + \sin^{-1}\left(\sqrt{1-x^2}\right)$.

a. Find $f(0)$, $f\left(\dfrac{1}{2}\right)$ and $f(1)$. 2 marks

b. Show that $f'(x) = 0$ for $0 < x < 1$, and hence write down the rule of f for $0 \leq x \leq 1$. 3 marks

c. Find $f\left(-\dfrac{1}{2}\right)$ and $f(-1)$. What does this tell you about your answer to part **b.**? 2 marks

d. Find $f'(x)$ for $-1 < x < 0$, and hence write f as a hybrid function. 3 marks

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e. State the range of f. 1 mark

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 2C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

The maximal domain of $y = \cos^{-1}\left(\frac{1}{x}\right)$ is

A. $[-1,0)\cup(0,1]$ **B.** $R\setminus\{0\}$ **C.** $(-\infty,-1]\cup[1,\infty)$ **D.** $[1,\infty)$

Answer: (A) (B) (C) (D)

Question 2C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

$\tan^{-1}(2)+\tan^{-1}(3)$ is equal to

A. $\frac{\pi}{2}$ **B.** $-\frac{\pi}{4}$ **C.** $\frac{3\pi}{4}$ **D.** $\tan^{-1}(5)$

Answer: (A) (B) (C) (D)

Question 2C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The graph of $y = a\sec(x)+b$ has turning points at $(0,3)$ and $(\pi,-1)$. The values of a and b are

A. $a=3, b=0$ **B.** $a=2, b=1$ **C.** $a=1, b=2$ **D.** $a=-2, b=1$

Answer: (A) (B) (C) (D)

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Circular Functions – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

2A.1 [2.1] **a.** $\sec\left(\frac{5\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$ **b.** $\cot\left(-\frac{\pi}{4}\right) = -1$ **c.** $\operatorname{cosec}\left(\frac{7\pi}{6}\right) = -2$ **d.** see Solutions **e.** $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ (the range of $\cos^{-1}$ is $[0,\pi]$) **f.** see Solutions

2A.2 [2.8] **a.** $x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$ **b.** $x = \frac{\pi}{3},\ \frac{2\pi}{3}$ **c.** $x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}$ **d.** $x = \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}$

2A.3 [2.7] **a.** LHS $= \tan(x)$ **b.** LHS $= \cos(2x)\times 1$ **c.** $\sin(2x+x) = 3\sin x - 4\sin^3 x$ **d.** see Solutions

2A.4 [2.4] **a.** shown in the Solutions **b.** $\sin\left(2\tan^{-1}(x)\right) = \frac{2x}{1+x^2}$ **c.** On this domain x^2-1 takes every value in $[-1,1]$, so the range is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ **d.** $x = \frac{1}{2}$

2B.1 [2.4] **a.** Range $(0,2\pi)$; asymptotes $y=0$ and $y=2\pi$ **b.** $f^{-1}\colon (0,2\pi)\to R,\ f^{-1}(x) = 1+\tan\left(\frac{x-\pi}{2}\right)$ **c.** explanation in the Solutions **d.** The point is $(5.879,\ 5.879)$

2B.2 [2.4] **a.** see Solutions **b.** shown in the Solutions **c.** see Solutions **d.** see Solutions **e.** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Section C (multiple choice): 1 C 2 C 3 B

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3 Vectors, Lines and Planes

Section A – Technology free

You may use the formula sheet.

Question 3A.1* (7 marks) ☐ Done ☐ Marked ☐ Redo

Let $\underset{\sim}{a} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{b} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$.

a. Find $|\underset{\sim}{a}|$. 1 mark

b. Find the unit vector in the direction of $\underset{\sim}{b}$. 1 mark

c. Find $\underset{\sim}{a} \cdot \underset{\sim}{b}$. 1 mark

d. Find the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, in exact form. 1 mark

e. Find the vector resolute of $\underset{\sim}{a}$ in the direction of $\underset{\sim}{b}$. 2 marks

f. Hence find the vector resolute of $\underset{\sim}{a}$ perpendicular to $\underset{\sim}{b}$. 1 mark

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Question 3A.2** (9 marks) ☐ Done ☐ Marked ☐ Redo

The line ℓ passes through $A(1,-2,3)$ and $B(3,0,2)$.

a. Find a vector equation of ℓ. 2 marks

b. Write ℓ in Cartesian form. 1 mark

c. Determine whether the point $(5,2,2)$ lies on ℓ. 1 mark

d. Find the coordinates of the point where ℓ crosses the xy-plane. 2 marks

e. Find the coordinates of the point on ℓ closest to the origin. 3 marks

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Question 3A.3*** (8 marks) ☐ Done ☐ Marked ☐ Redo

Consider the planes $\Pi_1 : x+y-z=1$ and $\Pi_2 : 2x-y+z=5$.

a. Show that Π_1 and Π_2 are perpendicular. 1 mark

b. Find a vector equation of the line of intersection of Π_1 and Π_2. 3 marks

c. Show that the line $\underset{\sim}{r}(s) = (\underset{\sim}{i}+\underset{\sim}{j}+\underset{\sim}{k})+s(\underset{\sim}{i}+2\underset{\sim}{j}+3\underset{\sim}{k})$ lies in Π_1. 2 marks

d. The line $\underset{\sim}{r}(s) = (\underset{\sim}{j}+2\underset{\sim}{k})+s(\underset{\sim}{i}+k\underset{\sim}{j}+2\underset{\sim}{k})$ is parallel to Π_2. Find k, and show that the line does not lie in Π_2. 2 marks

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Question 3A.4**** (10 marks) ☐ Done ☐ Marked ☐ Redo

In triangle OAB, $\overrightarrow{OA} = \underset{\sim}{a}$ and $\overrightarrow{OB} = \underset{\sim}{b}$. M is the midpoint of OA, and N is the point on AB such that $AN : NB = 1 : 2$. The lines ON and BM meet at P.

a. Express $\overrightarrow{ON}$ and $\overrightarrow{BM}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. 2 marks

b. By writing $\overrightarrow{OP}$ in two different ways, find $\overrightarrow{OP}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$. 4 marks

c. Find the ratios $OP : PN$ and $BP : PM$. 2 marks

d. Find the ratio of the area of triangle OPB to the area of triangle OAB. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 3B.1*** (10 marks) ☐ Done ☐ Marked ☐ Redo

Consider the points $A(2,1,0)$, $B(4,3,1)$ and $C(c,0,3)$, where $c \in R$.

a. Find c such that $\angle ABC = 90°$. 2 marks

b. Find the area of triangle ABC. 2 marks

c. Find the coordinates of D such that $ABCD$ is a rectangle. 2 marks

d. Find the Cartesian equation of the plane containing $ABCD$. 2 marks

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e. Find the acute angle between this plane and the xy-plane, correct to the nearest tenth of a degree. 2 marks

Question 3B.2***** (10 marks) ☐ Done ☐ Marked ☐ Redo

The line ℓ has equation $\underset{\sim}{r}(t) = (\underset{\sim}{i} + 2\underset{\sim}{k}) + t(2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k})$, and the plane Π_0 has equation $x + 2y + 2z = 0$. Let $\underset{\sim}{d} = 2\underset{\sim}{i} - \underset{\sim}{j} + 2\underset{\sim}{k}$ and $\underset{\sim}{m} = \underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k}$. The aim is to find the plane that contains ℓ and makes the smallest possible acute angle with Π_0.

a. A plane Π with normal $\underset{\sim}{n}$ contains ℓ. Explain why $\underset{\sim}{n} \cdot \underset{\sim}{d} = 0$. 1 mark

b. Find the acute angle between ℓ and Π_0, in degrees correct to two decimal places. 2 marks

c. Let $\underset{\sim}{n} = \underset{\sim}{m} - \lambda\underset{\sim}{d}$. Find the value of λ for which $\underset{\sim}{n}$ is perpendicular to $\underset{\sim}{d}$, and hence show that the plane with this normal that contains ℓ is $x + 22y + 10z = 21$. 3 marks

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d. Find the acute angle between this plane and Π_0, in degrees correct to two decimal places. 2 marks

e. Explain why no plane that contains ℓ can make a smaller angle with Π_0 than your answer to part **b.**, and hence why the plane in part **c.** is the one required. 2 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 3C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

The planes $x+2y-z=3$ and $2x+4y+kz=1$ are parallel when k equals

A. -2 **B.** 2 **C.** $-\dfrac{1}{2}$ **D.** 1

Answer: (A) (B) (C) (D)

Question 3C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

Let $\underset{\sim}{s}=3\,\underset{\sim}{i}-4\,\underset{\sim}{k}$ and let $\underset{\sim}{r}$ be a vector with $|\underset{\sim}{r}|=25$. The vector resolute of $\underset{\sim}{r}$ in the direction of $\underset{\sim}{s}$ is $9\,\underset{\sim}{i}-12\,\underset{\sim}{k}$. The scalar resolute of $\underset{\sim}{s}$ in the direction of $\underset{\sim}{r}$ is

A. 3 **B.** 5 **C.** 15 **D.** $\dfrac{3}{5}$

Answer: (A) (B) (C) (D)

Question 3C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The plane $2x+y-z=5$ is moved, without being tilted, until its equation is $2x+y-z=-1$. The plane has moved

A. $\sqrt{6}$ units in the direction of $2\,\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$

B. $\sqrt{6}$ units in the direction of $-2\,\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$

C. 6 units in the direction of $2\,\underset{\sim}{i}+\underset{\sim}{j}-\underset{\sim}{k}$

D. 6 units in the direction of $-2\,\underset{\sim}{i}-\underset{\sim}{j}+\underset{\sim}{k}$

Answer: (A) (B) (C) (D)

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Vectors, Lines and Planes – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

3A.1 [3.1] **a.** $|\underset{\sim}{a}| = 3$ **b.** $|\underset{\sim}{b}| = 3$, so $\hat{\underset{\sim}{b}} = \frac{1}{3}(\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$ **c.** $\underset{\sim}{a} \cdot \underset{\sim}{b} = 4$ **d.** $\theta = \cos^{-1}\left(\frac{4}{9}\right)$ **e.** $(\underset{\sim}{a} \cdot \hat{\underset{\sim}{b}})\hat{\underset{\sim}{b}} = \frac{4}{9}(\underset{\sim}{i} + 2\underset{\sim}{j} + 2\underset{\sim}{k})$ **f.** $\underset{\sim}{a} - \frac{4}{9}\underset{\sim}{b} = \frac{1}{9}(14\underset{\sim}{i} - 17\underset{\sim}{j} + 10\underset{\sim}{k})$

3A.2 [3.9] **a.** $\overrightarrow{AB} = 2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}$, so $\underset{\sim}{r}(t) = (\underset{\sim}{i} - 2\underset{\sim}{j} + 3\underset{\sim}{k}) + t(2\underset{\sim}{i} + 2\underset{\sim}{j} - \underset{\sim}{k}),\ t \in R$ **b.** $\frac{x-1}{2} = \frac{y+2}{2} = \frac{z-3}{-1}$ **c.** Not on ℓ **d.** $(7,4,0)$ **e.** see Solutions

3A.3 [3.10] **a.** shown in the Solutions **b.** $\underset{\sim}{r}(t) = (2\underset{\sim}{i} - \underset{\sim}{j}) + t(\underset{\sim}{j} + \underset{\sim}{k}),\ t \in R$ **c.** shown in the Solutions **d.** The point $(0,1,2)$ gives $0 - 1 + 2 = 1 \neq 5$, so the line is not in Π_2

3A.4 [3.7] **a.** $\overrightarrow{ON} = \underset{\sim}{a} + \frac{1}{3}(\underset{\sim}{b} - \underset{\sim}{a}) = \frac{2}{3}\underset{\sim}{a} + \frac{1}{3}\underset{\sim}{b}$, $\overrightarrow{BM} = \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}$ **b.** $\overrightarrow{OP} = \frac{2}{5}\underset{\sim}{a} + \frac{1}{5}\underset{\sim}{b}$ **c.** $OP : PN = 3:2,\ BP : PM = 4:1$ **d.** see Solutions

3B.1 [3.4] **a.** $c = 6$ **b.** see Solutions **c.** $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{BC} = (2,1,0) + (2,-3,2)$, so $D(4,-2,2)$ **d.** $7x - 2y - 10z = 12$ **e.** $\theta \approx 36.1°$

3B.2 [3.10] **a.** see Solutions **b.** $\phi = 26.39°$ **c.** $x + 22y + 10z = 21$ **d.** $\theta = 26.39°$, the same as part **b.** **e.** explanation in the Solutions

Section C (multiple choice): 1 A 2 A 3 B

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4 Complex Numbers

Section A – Technology free

You may use the formula sheet.

Question 4A.1* (7 marks) ☐ Done ☐ Marked ☐ Redo

a. Express $-1+\sqrt{3}\,i$ in polar form, using the principal argument. 2 marks

b. Express $4\,\text{cis}\left(-\dfrac{3\pi}{4}\right)$ in Cartesian form. 1 mark

c. Let $z=2\,\text{cis}\left(\dfrac{\pi}{3}\right)$ and $w=\sqrt{2}\,\text{cis}\left(-\dfrac{\pi}{4}\right)$. Find zw and $\dfrac{z}{w}$ in polar form. 2 marks

d. Find $(1+i)^8$. 2 marks

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Question 4A.2** (8 marks) ☐ Done ☐ Marked ☐ Redo

a. Solve $z^2+4z+13=0$ for $z \in C$. 2 marks

b. Given that $1-2i$ is a root of $P(z)=z^3-z^2+3z+5$, find the other roots of $P(z)=0$. 3 marks

c. By letting $z=a+bi$, solve $z^2=-3+4i$ for $z \in C$. 3 marks

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Question 4A.3**** (9 marks) ☐ Done ☐ Marked ☐ Redo

Let $\omega = \text{cis}\left(\frac{2\pi}{5}\right)$.

a. Show that $1+\omega+\omega^2+\omega^3+\omega^4=0$. 2 marks

b. Hence show that $\cos\left(\frac{2\pi}{5}\right)+\cos\left(\frac{4\pi}{5}\right)=-\frac{1}{2}$. 3 marks

c. Let $z=\text{cis}(\theta)$. Show that $z^n+z^{-n}=2\cos(n\theta)$ for $n\in N$. 1 mark

d. Solve $z^4-2z^2+4=0$ for $z\in C$, giving your answers in Cartesian form. 3 marks

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Question 4A.4*** (5 marks) ☐ Done ☐ Marked ☐ Redo

a. By writing $\text{cis}\left(\frac{5\pi}{12}\right) = \text{cis}\left(\frac{\pi}{4}\right)\text{cis}\left(\frac{\pi}{6}\right)$, find the exact value of $\cos\left(\frac{5\pi}{12}\right)$. 2 marks

b. Find the square roots of $1-\sqrt{3}i$ in Cartesian form. 3 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 4B.1***** (11 marks) ☐ Done ☐ Marked ☐ Redo

a. Let $z \in C$ with $|z| = 1$ and $z \neq 1$. Show that $\dfrac{z+1}{z-1}$ is purely imaginary (or zero). 3 marks

b. Interpret the result of part **a.** geometrically, in terms of the points -1, 1 and z on an Argand diagram. 2 marks

c. By letting $w = \dfrac{z+1}{z-1}$, solve $(z+1)^4 = (z-1)^4$ for $z \in C$. 3 marks

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d. Show that for any integer $n \geq 2$ the solutions of $(z+1)^n = (z-1)^n$ are $z = -i\cot\left(\dfrac{k\pi}{n}\right)$, $k = 1, 2, \ldots, n-1$, and state what these solutions have in common. 3 marks

Question 4B.2*** (14 marks) ☐ Done ☐ Marked ☐ Redo

a. The line $\text{Re}(z) = 2$ cuts the circle $|z| = 4$ at the points P and Q. Find P and Q in Cartesian form, and show that the chord PQ subtends an angle of $\dfrac{2\pi}{3}$ at the centre of the circle. 2 marks

b. Hence find the exact area of the minor segment of the circle cut off by the line, that is, the area of $\{z \in C : |z| \leq 4\} \cap \{z \in C : \text{Re}(z) \geq 2\}$. 2 marks

c. The ray $\text{Arg}(z) = \dfrac{\pi}{3}$ meets the circle $|z-2| = 2$ at the point A. Find A in Cartesian form. 2 marks

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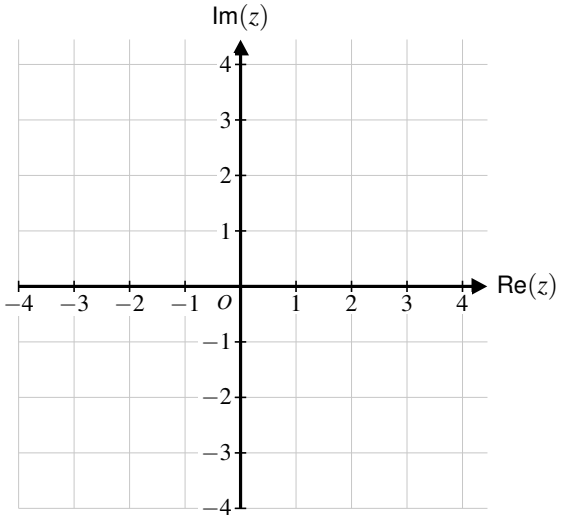
d. The circle $|z-2|=2$ passes through the origin O. Find the exact area of the minor segment of this circle cut off by the chord OA. 3 marks

e. Sketch $\{z \in C : |z| \leq 2\sqrt{2}\} \cap \{z \in C : |z-2-2i| \leq |z|\}$. 2 marks



f. Find the exact area of the region sketched in part **e.** 3 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 4C.1*** (1 mark) ☐ Done ☐ Marked ☐ Redo

Let $z = b - bi$, where $b > 0$. Then z^n, $n \in Z$, is purely imaginary exactly when

A. $n = 4k, k \in Z$ **B.** $n = 2k+1, k \in Z$ **C.** $n = 4k+2, k \in Z$ **D.** $n = 8k+2, k \in Z$

Answer: (A) (B) (C) (D)

Question 4C.2** (1 mark) ☐ Done ☐ Marked ☐ Redo

The rays $\text{Arg}(z) = \dfrac{\pi}{3}$ and $\text{Arg}(z-1) = \dfrac{2\pi}{3}$ intersect at

A. $\dfrac{1}{2} + \dfrac{\sqrt{3}}{2}i$ **B.** $1 + \sqrt{3}i$ **C.** $\dfrac{\sqrt{3}}{2} + \dfrac{1}{2}i$ **D.** $\sqrt{3}i$

Answer: (A) (B) (C) (D)

Question 4C.3**** (1 mark) ☐ Done ☐ Marked ☐ Redo

The number of distinct solutions of $z^3 = 4\bar{z}$, where $z \in C$, is

A. 3 **B.** 4 **C.** 5 **D.** 6

Answer: (A) (B) (C) (D)

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Complex Numbers – Short answers

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4A.1 [4.3] **a.** $2\operatorname{cis}\left(\dfrac{2\pi}{3}\right)$ **b.** $4\left(\cos\left(-\frac{3\pi}{4}\right)+i\sin\left(-\frac{3\pi}{4}\right)\right)=-2\sqrt{2}-2\sqrt{2}\,i$ **c.** see Solutions **d.** $1+i=\sqrt{2}\operatorname{cis}\frac{\pi}{4}$, so $(1+i)^8=(\sqrt{2})^8\operatorname{cis}(2\pi)=16$

4A.2 [4.5] **a.** $z=-2\pm 3i$ **b.** $P(z)=(z^2-2z+5)(z+1)$ (by inspection or division), so the roots are $1\pm 2i$ and -1 **c.** see Solutions

4A.3 [4.4] **a.** shown in the Solutions **b.** shown in the Solutions **c.** $z^n+z^{-n}=2\cos(n\theta)$, since the imaginary parts cancel **d.** $z=\pm\left(\dfrac{\sqrt{6}}{2}+\dfrac{\sqrt{2}}{2}i\right),\ z=\pm\left(\dfrac{\sqrt{6}}{2}-\dfrac{\sqrt{2}}{2}i\right)$

4A.4 [4.3] **a.** see Solutions **b.** see Solutions

4B.1 [4.2] **a.** a real multiple of i (zero when $z=-1$) **b.** explanation in the Solutions **c.** $z=0,\ i,\ -i$ **d.** shown in the Solutions

4B.2 [4.6] **a.** half the angle is $\dfrac{\pi}{3}$ and the chord subtends $\dfrac{2\pi}{3}$ **b.** see Solutions **c.** $A=1+\sqrt{3}i$ **d.** Area $=\dfrac{2\pi}{3}-\sqrt{3}$ **e.** sketch in the Solutions **f.** Area $=\dfrac{8\pi}{3}-2\sqrt{3}$

Section C (multiple choice): 1 C 2 A 3 C

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5 Differential Calculus and Sketching

Section A – Technology free

You may use the formula sheet.

Question 5A.1* (6 marks) ☐ Done ☐ Marked ☐ Redo

Differentiate each of the following with respect to x.

a. $\sin^{-1}(3x)$ 1 mark

b. $\tan^{-1}\left(\frac{x}{2}\right)$ 1 mark

c. $x\cos^{-1}(x)$ 2 marks

d. $\sec(2x)$ 1 mark

e. $\cot(x)$ 1 mark

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Question 5A.2** (6 marks) ☐ Done ☐ Marked ☐ Redo

a. Let $f(x) = \tan^{-1}(x)$. Find $f''(x)$ and hence find the coordinates of the point of inflection of the graph of f. 3 marks

b. Let $g(x) = x^4 - 4x^3$. Find the coordinates of the points of inflection of the graph of g, and state whether each is a stationary point of inflection. 3 marks

Question 5A.3**** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Let $f(x) = \sin^{-1}(x) + \cos^{-1}(x)$ for $x \in (-1, 1)$. Show that $f'(x) = 0$, and hence find $f(x)$. 2 marks

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b. Let $y = \tan^{-1}\left(\dfrac{1+x}{1-x}\right)$, $x < 1$. Show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$. 3 marks

c. Hence express $\tan^{-1}\left(\dfrac{1+x}{1-x}\right)$ in terms of $\tan^{-1}(x)$ for $x < 1$. 2 marks

d. For the circle $x^2 + y^2 = 25$, show that $\dfrac{d^2y}{dx^2} = -\dfrac{25}{y^3}$. 3 marks

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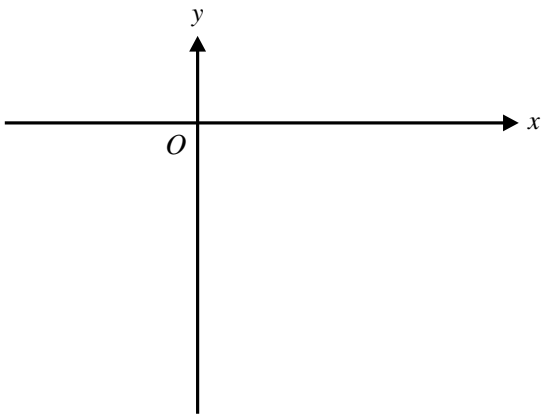
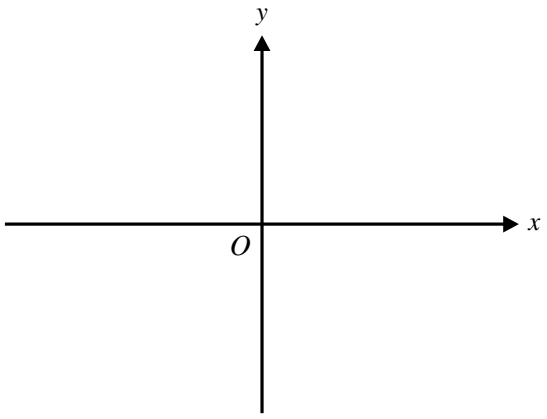
Question 5A.4*** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Sketch $\dfrac{(x-1)^2}{9}+\dfrac{(y+2)^2}{4}=1$, labelling the centre and the endpoints of the axes. 3 marks



b. Sketch $\dfrac{x^2}{4}-\dfrac{y^2}{9}=1$, labelling the vertices and the asymptotes. 3 marks



c. Find the Cartesian equation of the curve $x=\sec(t)$, $y=2\tan(t)$, $t\in\left(-\dfrac{\pi}{2},\dfrac{\pi}{2}\right)$, and state which part of the graph it traces. 2 marks

d. Find the centre and radius of the circle $x^2+y^2-6x+4y-12=0$. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 5B.1*** (7 marks) ☐ Done ☐ Marked ☐ Redo

A curve has equation $x^3 + y^3 = 6xy$.

a. Find $\dfrac{dy}{dx}$ in terms of x and y. 2 marks

b. Find the equation of the tangent to the curve at $(3,3)$. 2 marks

c. Find the coordinates of the point, other than the origin, where the tangent to the curve is horizontal. Give your answer correct to three decimal places. 3 marks

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Question 5B.2***** (11 marks) ☐ Done ☐ Marked ☐ Redo

A corridor 8 m wide meets a corridor 1 m wide at a right angle. A straight pole is carried horizontally around the corner. At the tightest moment the pole touches the inside corner and both outside walls, making an angle θ with the outside wall of the wide corridor, where $0 < \theta < \dfrac{\pi}{2}$.

a. Show that the length of a pole that just touches all three is $L(\theta) = 8\sec(\theta) + \operatorname{cosec}(\theta)$. 2 marks

b. Find $L'(\theta)$. 2 marks

c. Show that $L'(\theta) = 0$ when $\tan(\theta) = \dfrac{1}{2}$, and find the exact value of L there. 3 marks

d. Is this value a maximum or a minimum of L? Explain what it means for the longest pole that can be carried around the corner. 2 marks

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e. The narrow corridor is widened to w metres so that a 13 m pole just fits. Find w, correct to two decimal places.

2 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 5C.1*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The graph of $y = \dfrac{x^2+px+1}{x-1}$, where $p \in R$, has no turning points exactly when

A. $p < 2$ **B.** $p \leq -2$ **C.** $p > 2$ **D.** $p > -2$

Answer: (A) (B) (C) (D)

Question 5C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The graph of $f(x) = ax^4 + 2x^3 + 3x^2$, $a \in R$, has two points of inflection when

A. $a < \dfrac{1}{2}$ **B.** $a < \dfrac{1}{2}$ and $a \neq 0$ **C.** $a > \dfrac{1}{2}$ **D.** $a \neq 0$

Answer: (A) (B) (C) (D)

Question 5C.3** (1 mark) ☐ Done ☐ Marked ☐ Redo

The graph of $y = \dfrac{1}{x^2-4x+3}$ has

A. a local minimum at $(2,-1)$

B. a local maximum at $(2,1)$

C. a local maximum at $(2,-1)$

D. a single vertical asymptote $x = 2$

Answer: (A) (B) (C) (D)

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Differential Calculus and Sketching – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

5A.1 [5.1] **a.** $\dfrac{3}{\sqrt{1-9x^2}}$ **b.** $\dfrac{1/2}{1+\frac{x^2}{4}} = \dfrac{2}{4+x^2}$ **c.** $\cos^{-1}(x) - \dfrac{x}{\sqrt{1-x^2}}$ **d.** $2\sec(2x)\tan(2x)$ **e.** $-\operatorname{cosec}^2(x)$

5A.2 [5.2] **a.** see Solutions **b.** $(0,0)$ is a stationary point of inflection; $(2,-16)$ is a non-stationary point of inflection

5A.3 [5.1] **a.** $f(x) = \dfrac{\pi}{2}$ **b.** $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$ **c.** see Solutions **d.** see Solutions

5A.4 [5.10] **a.** sketch in the Solutions **b.** sketch in the Solutions **c.** $x^2 - \dfrac{y^2}{4} = 1$, the right branch only ($x \geq 1$) **d.** Centre $(3,-2)$, radius 5

5B.1 [5.3] **a.** $\dfrac{dy}{dx} = \dfrac{2y-x^2}{y^2-2x}$ **b.** $\dfrac{dy}{dx} = \dfrac{6-9}{9-6} = -1$, so $y = -x+6$ **c.** $(2.520,\ 3.175)$

5B.2 [5.1] **a.** shown in the Solutions **b.** $L'(\theta) = 8\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta$ **c.** shown in the Solutions **d.** see Solutions **e.** $w = 1.89$ m

Section C (multiple choice): 1 B 2 B 3 C

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6 Integration Techniques

Section A – Technology free

You may use the formula sheet.

Question 6A.1* (6 marks) ☐ Done ☐ Marked ☐ Redo

Find each of the following.

a. $\displaystyle\int \frac{1}{\sqrt{9-x^2}}\,dx$ 1 mark

b. $\displaystyle\int \frac{4}{4+x^2}\,dx$ 1 mark

c. $\displaystyle\int \sec^2(3x)\,dx$ 1 mark

d. $\displaystyle\int \frac{-2}{\sqrt{1-4x^2}}\,dx$ 1 mark

e. $\displaystyle\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}}\,dx$ 1 mark

f. $\displaystyle\int_0^{\sqrt{3}} \frac{1}{1+x^2}\,dx$ 1 mark

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Question 6A.2** (6 marks) ☐ Done ☐ Marked ☐ Redo

a. Using the substitution $u = x - 1$, find $\int x\sqrt{x-1}\,dx$. 3 marks

b. Using a suitable substitution, evaluate $\int_0^3 \frac{x}{\sqrt{x+1}}\,dx$. 3 marks

Question 6A.3*** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Find $\int \frac{5}{(x-2)(x+3)}\,dx$. 3 marks

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b. Express $\dfrac{3x+1}{(x-1)(x+1)^2}$ in partial fractions. 3 marks

c. Hence evaluate $\displaystyle\int_2^3 \frac{3x+1}{(x-1)(x+1)^2}\,dx$. 2 marks

d. Find $\displaystyle\int \frac{2x+3}{x^2+1}\,dx$. 2 marks

Question 6A.4**** (10 marks) ☐ Done ☐ Marked ☐ Redo

a. Show that $1+\sin(2x) = \big(\sin(x)+\cos(x)\big)^2$. 1 mark

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b. Hence evaluate $\int_0^{\pi} \sqrt{1+\sin(2x)}\,dx$. 4 marks

c. Evaluate $\int_0^{2\pi} \sqrt{1+\cos(x)}\,dx$. 3 marks

d. A student writes $\int_0^{\pi} \sqrt{1+\sin(2x)}\,dx = \Big[-\cos(x)+\sin(x)\Big]_0^{\pi} = 2$. Explain the mistake, and state the largest value of b for which this method gives the correct value of $\int_0^{b} \sqrt{1+\sin(2x)}\,dx$. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 6B.1*** (6 marks) ☐ Done ☐ Marked ☐ Redo

Let $f(x) = \dfrac{4}{x^2(x+2)}$ for $x > 0$.

a. Express $f(x)$ in partial fractions. 2 marks

b. Show that $\displaystyle\int_1^2 f(x)\,dx = 1 + \log_e\left(\frac{2}{3}\right)$. 2 marks

c. Find the value of $k > 1$ such that $\displaystyle\int_1^k f(x)\,dx = \frac{1}{2}$, correct to three decimal places. 2 marks

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Question 6B.2***** (12 marks) ☐ Done ☐ Marked ☐ Redo

Let $I_n = \int_0^{\frac{\pi}{2}} \sin^n(x)\,dx$ and $T_n = \int_0^{\frac{\pi}{4}} \tan^n(x)\,dx$, where n is a non-negative integer.

a. Use integration by parts to show that $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geq 2$. 4 marks

b. Hence find the exact values of I_5 and I_6. 3 marks

c. Show that $T_n + T_{n-2} = \dfrac{1}{n-1}$ for $n \geq 2$. 3 marks

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d. Hence find the exact value of T_4, and check it on your calculator. 2 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 6C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

Using $u = x+1$, $\int x(x+1)^9\,dx$ becomes

A. $\int (u-1)u^9\,du$ **B.** $\int (u+1)u^9\,du$ **C.** $\int u^{10}\,du$ **D.** $\int xu^9\,du$

Answer: (A) (B) (C) (D)

Question 6C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

Let $I_n = \int_0^{\pi/2} \cos^n(x)\,dx$, where $I_n = \dfrac{n-1}{n} I_{n-2}$ for $n \geq 2$. Then I_3 equals

A. $\dfrac{3\pi}{16}$ **B.** $\dfrac{1}{3}$ **C.** $\dfrac{2}{3}$ **D.** $\dfrac{\pi}{4}$

Answer: (A) (B) (C) (D)

Question 6C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

An antiderivative of $\log_2(x)$, for $x > 0$, is

A. $\dfrac{1}{x\log_e(2)}$ **B.** $x\log_2(x) - x$ **C.** $\log_e(2)\,(x\log_e(x) - x)$ **D.** $x\log_2(x) - \dfrac{x}{\log_e(2)}$

Answer: (A) (B) (C) (D)

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Integration Techniques – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

6A.1 [6.1] **a.** $\sin^{-1}\left(\frac{x}{3}\right)+c$ **b.** $2\int \frac{2}{2^2+x^2}\,dx = 2\tan^{-1}\left(\frac{x}{2}\right)+c$ **c.** $\frac{1}{3}\tan(3x)+c$ **d.** $-\int \frac{2}{\sqrt{1-(2x)^2}}\,dx = -\sin^{-1}(2x)+c$ (or $\cos^{-1}(2x)+c$) **e.** $\left[\sin^{-1}x\right]_0^{1/2} = \frac{\pi}{6}$ **f.** $\left[\tan^{-1}x\right]_0^{\sqrt{3}} = \frac{\pi}{3}$

6A.2 [6.4] **a.** $\int (u+1)u^{\frac{1}{2}}\,du = \frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+c$ **b.** see Solutions

6A.3 [6.5] **a.** $\log_e|x-2|-\log_e|x+3|+c = \log_e\left|\frac{x-2}{x+3}\right|+c$ **b.** $\frac{1}{x-1}-\frac{1}{x+1}+\frac{1}{(x+1)^2}$ **c.** $\left[\log_e(x-1)-\log_e(x+1)-\frac{1}{x+1}\right]_2^3 = \log_e\left(\frac{3}{2}\right)+\frac{1}{12}$ **d.** $\int\left(\frac{2x}{x^2+1}+\frac{3}{x^2+1}\right)dx = \log_e(x^2+1)+3\tan^{-1}(x)+c$

6A.4 [6.7] **a.** shown in the Solutions **b.** $=2\sqrt{2}$ **c.** see Solutions **d.** the largest value is $b=\frac{3\pi}{4}$

6B.1 [6.5] **a.** $\frac{4}{x^2(x+2)} = -\frac{1}{x}+\frac{2}{x^2}+\frac{1}{x+2}$ **b.** shown in the Solutions **c.** $k\approx 1.693$

6B.2 [6.6] **a.** shown in the Solutions **b.** see Solutions **c.** shown in the Solutions **d.** $T_0=\frac{\pi}{4}$, $T_2=1-\frac{\pi}{4}$, $T_4=\frac{1}{3}-T_2=\frac{\pi}{4}-\frac{2}{3}\approx 0.1187$

Section C (multiple choice): 1 A 2 C 3 D

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7 Applications of Integration

Section A – Technology free

You may use the formula sheet.

Question 7A.1* (5 marks) ☐ Done ☐ Marked ☐ Redo

Let $F(x) = \int_1^x \sqrt{t^3+1}\,dt$.

a. Find $F(1)$. 1 mark

b. Find $F'(2)$. 1 mark

c. Let $G(x) = \int_0^{x^2} \sin(t)\,dt$. Find $G'(x)$. 2 marks

d. Evaluate $\int_0^{\pi} |\cos(x)|\,dx$. 1 mark

Question 7A.2** (7 marks) ☐ Done ☐ Marked ☐ Redo

Find the exact volume of the solid formed when each region is rotated about the x-axis.

a. The region under $y = \sqrt{x}$ for $0 \leq x \leq 4$. 2 marks

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b. The region under $y = \sec(x)$ for $0 \leq x \leq \dfrac{\pi}{4}$. 2 marks

c. The region enclosed by $y = x^2$ and $y = 2x$. 3 marks

Question 7A.3*** (6 marks) ☐ Done ☐ Marked ☐ Redo

a. The curve $y = \sqrt{x}$, $0 \leq x \leq 2$, is rotated about the x-axis. Find the exact area of the surface formed. 3 marks

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b. Show that the surface area of a sphere of radius r is $4\pi r^2$. 3 marks

Question 7A.4**** (8 marks) ☐ Done ☐ Marked ☐ Redo

a. Let $F(x) = \int_0^x e^{-t^2}\,dt$. Explain why F is strictly increasing, and show that $F(-x) = -F(x)$. 2 marks

b. Let $H(x) = \int_x^{2x} \frac{1}{t}\,dt$ for $x > 0$. Show that $H'(x) = 0$ and explain what this tells you about H. 2 marks

c. Find the area of the region enclosed by $y = \sin^{-1}(x)$, the y-axis and the line $y = \frac{\pi}{2}$. 2 marks

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d. Hence find $\int_0^1 \sin^{-1}(x)\,dx$. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 7B.1*** (9 marks) ☐ Done ☐ Marked ☐ Redo

The following pseudocode is run.

```
define f(x) = sqrt(x) + 1
left ← 0
total ← 0
h ← 0.5
while left < 2
  r ← f(left + h/2)
  total ← total + π× r^2× h
  left ← left + h
end while
print total
```

a. Complete a table showing the values of `left`, `r` and `total` each time the loop runs, and state the value printed, correct to three decimal places. 3 marks

b. Describe the solid whose volume the algorithm approximates, write down a definite integral for the exact volume, and evaluate it. 3 marks

c. Find the percentage error of the printed value. 1 mark

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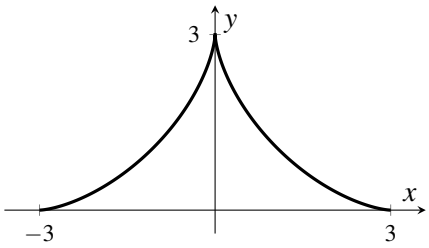
d. State one change to a single line that would make the approximation more accurate, and say how many times the loop would then run. 2 marks

Question 7B.2***** (13 marks) ☐ Done ☐ Marked ☐ Redo

A curve is defined by $x = 3\cos^3(t),\ y = 3\sin^3(t)$ for $0 \le t \le \pi$.



a. Find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. 2 marks

b. Show that $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = 9\,|\sin(t)\cos(t)|$. 3 marks

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c. Find the length of the curve. 3 marks

d. The curve is rotated about the x-axis. Find the exact area of the surface formed. 4 marks

e. A student leaves out the modulus signs in part **d.** State the answer the student gets, and explain why it cannot be correct. 1 mark

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 7C.1*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The length of the curve $y = \dfrac{e^x + e^{-x}}{2}$ from $x = 0$ to $x = \log_e 2$ is

A. $\dfrac{1}{2}$ **B.** $\dfrac{5}{4}$ **C.** $\dfrac{3}{4}$ **D.** $\log_e 2$

Answer: (A) (B) (C) (D)

Question 7C.2** (1 mark) ☐ Done ☐ Marked ☐ Redo

The curve $y = f(x)$, $a \leq x \leq b$ (with $a \geq 0$), is rotated about the y-axis. The surface area is

A. $2\pi \displaystyle\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx$ **B.** $\pi \displaystyle\int_a^b x^2\,dy$

C. $2\pi \displaystyle\int_a^b \sqrt{1+(f'(x))^2}\,dx$ **D.** $2\pi \displaystyle\int_a^b x\sqrt{1+(f'(x))^2}\,dx$

Answer: (A) (B) (C) (D)

Question 7C.3** (1 mark) ☐ Done ☐ Marked ☐ Redo

If $F(x) = \displaystyle\int_0^x \cos(t^2)\,dt$, then $F'(\sqrt{\pi})$ equals

A. -1 **B.** 1 **C.** 0 **D.** $\cos(\sqrt{\pi})$

Answer: (A) (B) (C) (D)

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Applications of Integration – Short answers

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7A.1 [7.1] **a.** $F(1) = 0$ **b.** By the fundamental theorem, $F'(x) = \sqrt{x^3+1}$, so $F'(2) = \sqrt{9} = 3$ **c.** $G'(x) = 2x\sin(x^2)$ **d.** $\displaystyle\int_0^{\pi/2} \cos x\,dx - \int_{\pi/2}^{\pi} \cos x\,dx = 2$

7A.2 [7.3] **a.** $\displaystyle\pi\int_0^4 x\,dx = 8\pi$ **b.** $\displaystyle\pi\int_0^{\pi/4} \sec^2 x\,dx = \pi$ **c.** $\displaystyle\pi\int_0^2 \left(4x^2 - x^4\right)dx = \frac{64\pi}{15}$

7A.3 [7.5] **a.** see Solutions **b.** shown in the Solutions

7A.4 [7.1] **a.** $= -F(x)$ **b.** $H'(x) = 0$, so H is constant ($H(x) = \log_e 2$) **c.** area $= 1$ **d.** see Solutions

7B.1 [7.3] **a.** see Solutions **b.** explanation in the Solutions **c.** $\dfrac{24.526 - 24.414}{24.414} \times 100\% \approx 0.46\%$ **d.** The loop would then run $\dfrac{2}{0.1} = 20$ times

7B.2 [7.4] **a.** $\dfrac{dx}{dt} = -9\cos^2(t)\sin(t)$, $\dfrac{dy}{dt} = 9\sin^2(t)\cos(t)$ **b.** shown in the Solutions **c.** $L = 9$ **d.** $S = \dfrac{108\pi}{5}$ **e.** see Solutions

Section C (multiple choice): 1 C 2 D 3 A

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8 Differential Equations

Section A – Technology free

You may use the formula sheet.

Question 8A.1* (5 marks) ☐ Done ☐ Marked ☐ Redo

a. Show that $y = e^{2x} + x$ is a solution of $\dfrac{dy}{dx} - 2y = 1 - 2x$. 2 marks

b. Show that $y = \sin(3x)$ is a solution of $\dfrac{d^2y}{dx^2} + 9y = 0$. 1 mark

c. Find the values of k for which $y = e^{kx}$ is a solution of $\dfrac{d^2y}{dx^2} - \dfrac{dy}{dx} - 6y = 0$. 2 marks

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Question 8A.2** (7 marks) ☐ Done ☐ Marked ☐ Redo

Solve each differential equation, giving y in terms of x.

a. $\dfrac{dy}{dx} = 2y, \quad y(0) = 3$ 2 marks

b. $\dfrac{dy}{dx} = 1 + y^2, \quad y(0) = 1$ 3 marks

c. $\dfrac{dy}{dx} = e^{-y}, \quad y(0) = 0$ 2 marks

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Question 8A.3*** (8 marks) ☐ Done ☐ Marked ☐ Redo

A cup of coffee at $90°$C is left in a room at $20°$C. Its temperature $T°$C after t minutes satisfies $\frac{dT}{dt} = -k(T-20)$, where $k > 0$.

a. Solve the differential equation to find T in terms of t and k. 2 marks

b. After 10 minutes the coffee is at $55°$C. Find the exact value of k. 2 marks

c. Find when the coffee is at $37.5°$C. 1 mark

d. A tank holds 100 L of water in which 10 kg of salt is dissolved. Pure water flows in at 5 L/min and the well-mixed solution flows out at 5 L/min. Write down and solve a differential equation for the amount of salt x kg in the tank after t minutes. 3 marks

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Question 8A.4**** (8 marks) ☐ Done ☐ Marked ☐ Redo

A population P of fish in a lake after t years satisfies $\dfrac{dP}{dt} = 0.2P\left(1 - \dfrac{P}{500}\right)$, with $P(0) = 100$.

a. Show that $\dfrac{1}{P\left(1-\frac{P}{500}\right)} = \dfrac{1}{P} + \dfrac{1}{500-P}$. 1 mark

b. Hence show that $P = \dfrac{500}{1+4e^{-0.2t}}$. 4 marks

c. State the limiting population. 1 mark

d. The population grows fastest when $P = 250$. Find the exact time at which this occurs. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 8B.1***** (14 marks) ☐ Done ☐ Marked ☐ Redo

A fish farm is modelled by $\dfrac{dP}{dt} = 0.2P\left(1 - \dfrac{P}{1000}\right) - 32$, where P is the number of fish after t years and 32 fish are removed each year.

a. Show that $\dfrac{dP}{dt} = -\dfrac{(P-200)(P-800)}{5000}$. 2 marks

b. State the two equilibrium populations, and describe what happens in the long run when $P(0) = 500$, when $P(0) = 900$ and when $P(0) = 150$. 3 marks

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c. For $P(0) = 500$, solve the differential equation to find P in terms of t, and find the time taken for the population to reach 700, correct to two decimal places. 5 marks

d. For $P(0) = 150$, write down a definite integral for the time taken for the fish to die out, and evaluate it correct to two decimal places. 2 marks

e. The number removed each year is changed from 32 to h. Find the largest value of h for which the population can survive. 2 marks

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Question 8B.2*** (8 marks) ☐ Done ☐ Marked ☐ Redo

A beam of length 4 m rests on supports at its two ends. Its height y metres above the supports, at a horizontal distance x metres from the left end, satisfies $\dfrac{d^2y}{dx^2} = \dfrac{4x - x^2}{100}$ with $y(0) = 0$ and $y(4) = 0$.

a. Find y in terms of x. 4 marks

b. Find the greatest distance that the beam sags below its supports. 2 marks

c. Find the angle that the beam makes with the horizontal at its left end, in degrees correct to two decimal places. 2 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 8C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

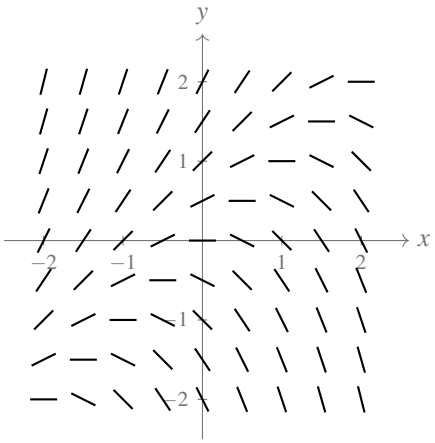
A 100 L tank holds brine containing 5 kg of salt. Brine with 0.2 kg/L flows in at 3 L/min and the mixture flows out at 3 L/min. If x kg is the salt in the tank,

A. $\dfrac{dx}{dt} = 0.2 - \dfrac{3x}{100}$ **B.** $\dfrac{dx}{dt} = 0.6 - \dfrac{3x}{100}$ **C.** $\dfrac{dx}{dt} = 0.6 - \dfrac{x}{100}$ **D.** $\dfrac{dx}{dt} = 0.6 - \dfrac{3x}{100+t}$

Answer: (A) (B) (C) (D)

Question 8C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

The slope field of a differential equation is shown.



The differential equation could be

A. $\dfrac{dy}{dx} = x + y$ **B.** $\dfrac{dy}{dx} = x - y$ **C.** $\dfrac{dy}{dx} = y - x$ **D.** $\dfrac{dy}{dx} = xy$

Answer: (A) (B) (C) (D)

Question 8C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

y is the solution of $\dfrac{dy}{dx} = 3x^2y^2$ with $y(0) = \dfrac{1}{8}$. The largest set of values of x for which this solution is defined is

A. $x \in R$ **B.** $x < 2$ **C.** $x \in R \setminus \{2\}$ **D.** $-2 < x < 2$

Answer: (A) (B) (C) (D)

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Differential Equations – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

8A.1 [8.1] **a.** shown in the Solutions **b.** shown in the Solutions **c.** $k = 3$ or $k = -2$

8A.2 [8.3] **a.** $y = 3e^{2x}$ **b.** $y = \tan\left(x + \frac{\pi}{4}\right)$ **c.** $y = \log_e(x+1)$

8A.3 [8.5] **a.** $T = 20 + 70e^{-kt}$ **b.** $k = \frac{\log_e 2}{10}$ **c.** $t = 20$ minutes **d.** $x = 10e^{-t/20}$

8A.4 [8.6] **a.** shown in the Solutions **b.** shown in the Solutions **c.** As $t \to \infty$, $e^{-0.2t} \to 0$, so $P \to 500$ **d.** $t = 10\log_e 2$ years

8B.1 [8.6] **a.** shown in the Solutions **b.** $\frac{dP}{dt} < 0$, so P falls, faster and faster, and the fish die out **c.** $t = 13.41$ years **d.** $t = 9.82$ years **e.** see Solutions

8B.2 [8.2] **a.** $y = \frac{8x^3 - x^4}{1200} - \frac{4x}{75}$ **b.** The beam sags $\frac{1}{15}$ m, about 6.7 cm **c.** see Solutions

Section C (multiple choice): 1 B 2 C 3 B

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9 Kinematics

Section A – Technology free

You may use the formula sheet.

Question 9A.1* (6 marks) ☐ Done ☐ Marked ☐ Redo

A particle moves in a straight line so that its position x m from the origin after t seconds is $x = t^3 - 6t^2 + 9t$, $t \geq 0$.

a. Find the velocity of the particle in terms of t. 1 mark

b. Find when the particle is at rest. 1 mark

c. Find the acceleration of the particle when $t = 2$. 1 mark

d. Find the distance travelled by the particle in the first 4 seconds. 2 marks

e. Find the average velocity of the particle over the first 4 seconds. 1 mark

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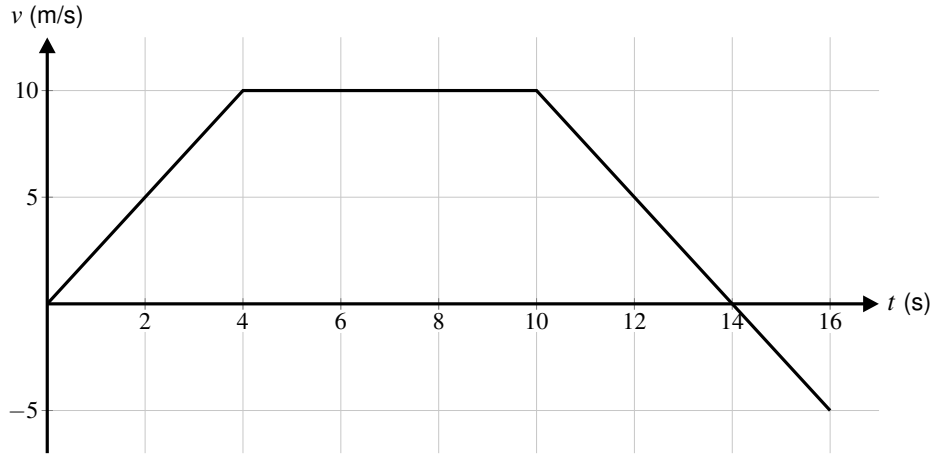
Question 9A.2** (5 marks) ☐ Done ☐ Marked ☐ Redo

The velocity–time graph of a particle moving in a straight line for $0 \leq t \leq 16$ is shown below.



a. Find the acceleration of the particle during the first 4 seconds. 1 mark

b. Find the displacement of the particle over the 16 seconds. 2 marks

c. Find the distance travelled over the 16 seconds. 1 mark

d. At what time is the particle furthest from its starting point? 1 mark

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Question 9A.3*** (6 marks) ☐ Done ☐ Marked ☐ Redo

A particle starts from rest at the origin and moves with acceleration $a = \dfrac{2}{(t+1)^2}$ m/s^2.

a. Find the velocity and position of the particle in terms of t. 3 marks

b. State the velocity that the particle approaches as $t \to \infty$. 1 mark

c. Show that $a = \dfrac{(2-v)^2}{2}$. 2 marks

Question 9A.4**** (8 marks) ☐ Done ☐ Marked ☐ Redo

A boat cuts its engine when travelling at u m/s. Its acceleration is then $a = -kv^2$, where $k > 0$.

a. Find v in terms of t. 2 marks

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b. Find v in terms of the distance x travelled since the engine was cut. 2 marks

c. Explain why, according to this model, the boat never stops but travels an unbounded distance. 2 marks

d. The boat's speed halves over the first 100 m. Find the exact value of k. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 9B.1***** (13 marks) ☐ Done ☐ Marked ☐ Redo

A skydiver steps out of a stationary helicopter 1000 m above the ground. For the first 10 seconds her acceleration is $a = 9.8 - 0.2v$, where v m/s is her downward speed. Her parachute then opens instantly, and from then on $a = 9.8 - 1.96v$.

a. Find v in terms of t for $0 \leq t \leq 10$, and her speed when the parachute opens, correct to two decimal places. 3 marks

b. Find the distance she falls in the first 10 seconds, correct to two decimal places. 2 marks

c. State her terminal speed once the parachute is open. 1 mark

d. Find v in terms of t for $t \geq 10$, and how long after the parachute opens her speed first drops to 6 m/s. 3 marks

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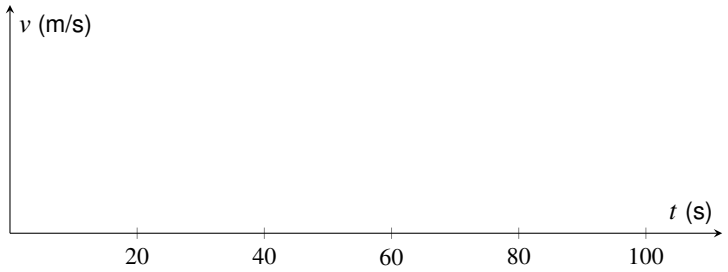
e. Find the total time she takes to reach the ground, correct to one decimal place. 3 marks

f. Find her speed when she lands. 1 mark

Question 9B.2*** (10 marks) ☐ Done ☐ Marked ☐ Redo

A train starts from rest at one station and accelerates uniformly for 20 seconds until it reaches a speed of V m/s. It travels at this speed for a while, then decelerates uniformly for 10 seconds and stops at the next station. The whole journey takes 100 seconds and covers 1700 m.

a. Sketch the velocity–time graph of the journey on the axes below. 2 marks



b. Find V. 3 marks

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c. Find the acceleration of the train in the first stage and in the last stage. 2 marks

d. Find the time at which the train is halfway between the stations. 2 marks

e. Find the average speed of the train for the journey. 1 mark

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 9C.1*** (1 mark) ☐ Done ☐ Marked ☐ Redo

A car travelling at 10 m/s hits a barrier and slows with acceleration $-5v^2$ m/s^2. The distance it travels while slowing to 1 m/s is closest to

A. 0.46 m **B.** 0.18 m **C.** 2.30 m **D.** 0.90 m

Answer: (A) (B) (C) (D)

Question 9C.2** (1 mark) ☐ Done ☐ Marked ☐ Redo

A particle moving with velocity 2 m/s has acceleration $a = -(4+v^2)$. The time it takes to come to rest is

A. $\dfrac{\pi}{4}$ **B.** $\dfrac{1}{2}$ **C.** $\dfrac{\pi}{2}$ **D.** $\dfrac{\pi}{8}$

Answer: (A) (B) (C) (D)

Question 9C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

A particle moves in a straight line with constant acceleration. It passes a point A at 2 m/s and later a point B at 14 m/s. Its speed, in m/s, at the point halfway between A and B is

A. 6 **B.** 8 **C.** 10 **D.** 12

Answer: (A) (B) (C) (D)

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Kinematics – Short answers

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9A.1 [9.1] **a.** $v = 3t^2 - 12t + 9$ **b.** $t = 1$ and $t = 3$ seconds **c.** $a = 6t - 12$, so at $t = 2$, $a = 0\,\text{m/s}^2$ **d.** Distance $= 12\,\text{m}$ **e.** $\dfrac{x(4) - x(0)}{4} = 1\,\text{m/s}$

9A.2 [9.3] **a.** Gradient $= \dfrac{10}{4} = 2.5\,\text{m/s}^2$ **b.** Signed area $= 95\,\text{m}$ **c.** $20 + 60 + 20 + 5 = 105\,\text{m}$ **d.** At $t = 14$, when the velocity changes from positive to negative

9A.3 [9.1] **a.** $x = 2t - 2\log_e(t+1)$ **b.** $v \to 2\,\text{m/s}$ **c.** shown in the Solutions

9A.4 [9.4] **a.** $v = \dfrac{u}{1+ukt}$ **b.** $v = ue^{-kx}$ **c.** explanation in the Solutions **d.** $k = \dfrac{\log_e 2}{100}$

9B.1 [9.4] **a.** $v = 49\left(1 - e^{-0.2t}\right)$, and $v(10) = 49\left(1 - e^{-2}\right) = 42.37\,\text{m/s}$ **b.** $\displaystyle\int_0^{10} 49\left(1 - e^{-0.2t}\right) dt = 278.16\,\text{m}$ **c.** $a = 0$ when $v = \dfrac{9.8}{1.96} = 5\,\text{m/s}$ **d.** $t - 10 = 1.85\,\text{s}$ **e.** Total time $= 150.6\,\text{s}$ **f.** $5 + 37.37e^{-1.96 \times 140.56} \approx 5.00\,\text{m/s}$ (she is at terminal speed)

9B.2 [9.3] **a.** sketch in the Solutions **b.** $V = 20\,\text{m/s}$ **c.** $1\,\text{m/s}^2$ and $-2\,\text{m/s}^2$ **d.** $t = 52.5\,\text{s}$ **e.** $\dfrac{1700}{100} = 17\,\text{m/s}$

Section C (multiple choice): 1 A 2 D 3 C

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10 Vector Calculus

Section A – Technology free

You may use the formula sheet.

Question 10A.1* (7 marks) ☐ Done ☐ Marked ☐ Redo

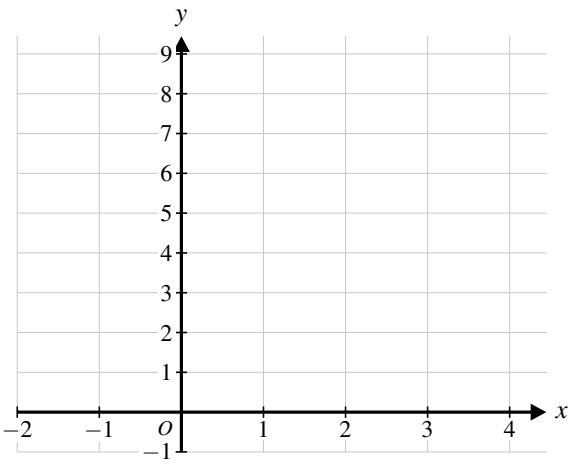
The position of a particle at time $t \geq 0$ is $\underset{\sim}{r}(t) = (t-1)\underset{\sim}{i} + t^2\underset{\sim}{j}$.

a. Find the Cartesian equation of the path, stating any restriction on x. 2 marks

b. Find the velocity and acceleration of the particle. 2 marks

c. Find the speed of the particle when $t = 1$. 1 mark

d. Sketch the path of the particle on the axes below, showing its starting point and direction of motion. 2 marks



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Question 10A.2** (6 marks) ☐ Done ☐ Marked ☐ Redo

A curve is defined by $x = t^2$, $y = t^3 - 3t$, $t \in R$.

a. Find $\dfrac{dy}{dx}$ in terms of t. 1 mark

b. Find the coordinates of the points on the curve where the tangent is horizontal. 2 marks

c. Find the coordinates of the point on the curve where the tangent is vertical. 1 mark

d. Find the equation of the tangent to the curve at the point where $t = 2$. 2 marks

Question 10A.3*** (5 marks) ☐ Done ☐ Marked ☐ Redo

A particle moves so that $\underset{\sim}{r}(t) = t^2 \underset{\sim}{i} + \dfrac{2}{3}t^3 \underset{\sim}{j}$, $t \geq 0$.

a. Find the distance travelled by the particle from $t = 0$ to $t = \sqrt{3}$. 3 marks

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b. Find the magnitude of the displacement over the same interval, and explain why it is less than your answer to part **a.** 2 marks

Question 10A.4**** (8 marks) ☐ Done ☐ Marked ☐ Redo

Two particles move so that $\underset{\sim}{r}_A(t) = (2t-1)\underset{\sim}{i} + \left(t^2-2\right)\underset{\sim}{j}$ and $\underset{\sim}{r}_B(t) = (t+1)\underset{\sim}{i} + (3t-4)\underset{\sim}{j}$, $t \geq 0$.

a. Show that the particles collide, and find when and where. 2 marks

b. The paths of the particles also cross at a second point. Find this point, and the time at which each particle passes through it. 3 marks

c. Find the distance between the particles at $t = 0$. 1 mark

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d. Find the cosine of the angle between the directions of motion of the particles when they collide. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 10B.1***** (13 marks) ☐ Done ☐ Marked ☐ Redo

Three drones P, Q and R fly along edges of a cube of side 4 units. For $0 \leq t \leq 4$ their position vectors are

$$\underset{\sim}{r}_P(t) = t\,\underset{\sim}{i}, \qquad \underset{\sim}{r}_Q(t) = 4\underset{\sim}{i} + t\,\underset{\sim}{j}, \qquad \underset{\sim}{r}_R(t) = 4\underset{\sim}{i} + 4\underset{\sim}{j} + t\,\underset{\sim}{k}.$$

a. Find the velocity of each drone, and the angle between the directions of motion of any two of them. 2 marks

b. Show that the distance d between P and Q satisfies $d^2 = 2t^2 - 8t + 16$, and find the minimum distance between them. 3 marks

c. Show that the area A of triangle PQR satisfies $A^2 = \frac{1}{4}(16-s)(16-3s)$, where $s = t(4-t)$. 4 marks

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d. Hence find the minimum area of triangle PQR and the time at which it occurs. 3 marks

e. State the maximum area of the triangle for $0 \leq t \leq 4$. 1 mark

Question 10B.2*** (10 marks) ☐ Done ☐ Marked ☐ Redo

An observer stands at the origin O on level ground. A drone has position

$$\underset{\sim}{r}(t) = 10t\,\underset{\sim}{i} + (60-5t)\,\underset{\sim}{j} + \left(20+4t-0.2t^2\right)\underset{\sim}{k} \text{ metres}, \quad 0 \leq t \leq 20,$$

where t is the time in seconds, $\underset{\sim}{i}$ and $\underset{\sim}{j}$ are horizontal and $\underset{\sim}{k}$ is vertically up.

a. Find the position of the drone when $t = 4$ and its distance from the observer, correct to one decimal place. 2 marks

b. Find the angle of elevation of the drone from the observer when $t = 4$, in degrees correct to one decimal place. 3 marks

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c. Find the angle of elevation of the drone from the observer at the instant it reaches its greatest height. 2 marks

d. Find the speed of the drone when $t = 4$, and the angle its direction of motion makes with the horizontal at that instant. 3 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 10C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

A projectile has position $\underset{\sim}{r}(t) = 20t\,\underset{\sim}{i} + (20t - 4.9t^2)\,\underset{\sim}{j}$ metres. Its maximum height is closest to

A. $20.4\,\text{m}$ **B.** $40.8\,\text{m}$ **C.** $81.6\,\text{m}$ **D.** $4.1\,\text{m}$

Answer: (A) (B) (C) (D)

Question 10C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

A ball is hit from a point 1 m above level ground. Its position t seconds later is $\underset{\sim}{r}(t) = 14t\,\underset{\sim}{i} + \left(1 + 9.8t - 4.9t^2\right)\,\underset{\sim}{j}$ metres, where $\underset{\sim}{i}$ is horizontal and $\underset{\sim}{j}$ is vertically up. The acute angle that the ball's path makes with the ground at the instant it lands is closest to

A. 35.0° **B.** 37.5° **C.** 52.5° **D.** 55.0°

Answer: (A) (B) (C) (D)

Question 10C.3*** (1 mark) ☐ Done ☐ Marked ☐ Redo

A particle has position $\underset{\sim}{r}(t) = k\sin(t)\,\underset{\sim}{i} + t^2\,\underset{\sim}{j}$, where k is a positive constant. Its acceleration is perpendicular to its velocity when $t = \dfrac{\pi}{4}$. The value of k is

A. $\sqrt{\dfrac{\pi}{2}}$ **B.** $\sqrt{\pi}$ **C.** 2 **D.** $\sqrt{2\pi}$

Answer: (A) (B) (C) (D)

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Vector Calculus – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

10A.1 [10.1] **a.** $y=(x+1)^2,\ x\geq -1$ **b.** $\underset{\sim}{\dot{r}}(t)=\underset{\sim}{i}+2t\,\underset{\sim}{j},\ \underset{\sim}{\ddot{r}}(t)=2\,\underset{\sim}{j}$ **c.** $|\underset{\sim}{i}+2\,\underset{\sim}{j}|=\sqrt{5}$ **d.** sketch in the Solutions

10A.2 [5.8] **a.** $\dfrac{dy}{dx}=\dfrac{3t^2-3}{2t}$ **b.** the points $(1,-2)$ and $(1,2)$ **c.** the point $(0,0)$ **d.** $y=\dfrac{9}{4}x-7$

10A.3 [10.3] **a.** $\displaystyle\int_0^{\sqrt{3}} 2t\sqrt{1+t^2}\,dt=\frac{14}{3}$ **b.** see Solutions

10A.4 [10.4] **a.** shown in the Solutions **b.** A passes through it at $t=4$ and B at $t=6$, so they cross without colliding **c.** distance $=2\sqrt{2}$ **d.** $\cos\theta=\dfrac{7\sqrt{2}}{10}$

10B.1 [10.2] **a.** the directions are at 90° to each other **b.** shown in the Solutions **c.** shown in the Solutions **d.** minimum area $2\sqrt{3}$ square units at $t=2$ **e.** $A^2=64$, so the maximum area is 8

10B.2 [10.1] **a.** Distance $=65.4$ m **b.** $\theta=30.1^\circ$ **c.** $\cos\theta=\dfrac{10\,100}{\sqrt{11\,700}\sqrt{10\,100}}=0.9291$, so $\theta=21.7^\circ$ **d.** $\alpha=12.1^\circ$ above the horizontal (the $\underset{\sim}{k}$ component is positive, so it is climbing)

Section C (multiple choice): 1 A 2 B 3 D

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11 Probability and Statistics

Section A – Technology free

You may use the formula sheet.

Question 11A.1* (6 marks) ☐ Done ☐ Marked ☐ Redo

X and Y are independent random variables with $\mathrm{E}(X) = 5$, $\mathrm{Var}(X) = 4$, $\mathrm{E}(Y) = 3$ and $\mathrm{Var}(Y) = 9$.

a. Find $\mathrm{E}(2X - 3)$ and $\mathrm{Var}(2X - 3)$. 2 marks

b. Find $\mathrm{E}(X - Y)$ and $\mathrm{Var}(X - Y)$. 2 marks

c. X_1, X_2 and X_3 are independent observations of X. Find $\mathrm{Var}(X_1 + X_2 + X_3)$ and explain why it is not equal to $\mathrm{Var}(3X)$. 2 marks

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Question 11A.2** (6 marks) ☐ Done ☐ Marked ☐ Redo

A random sample of 100 batteries has a mean lifetime of 52 hours. The population standard deviation is known to be 10 hours. Use $z = 1.96$ for 95% confidence.

a. Find a 95% confidence interval for the mean lifetime μ. 2 marks

b. Explain what is meant by "95% confidence" in this context. 1 mark

c. Find the smallest sample size needed for the margin of error of a 95% confidence interval to be at most 1 hour. 2 marks

d. State how a 99% confidence interval from the same sample would compare with the interval in part **a.** 1 mark

Question 11A.3**** (8 marks) ☐ Done ☐ Marked ☐ Redo

A random variable X has the probability distribution below.

x	0	1	2
$\Pr(X = x)$	0.2	0.5	0.3

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a. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. 3 marks

b. Y is the sum of 100 independent observations of X. State the approximate distribution of Y, with its mean and standard deviation. 2 marks

c. Use the fact that about 95% of a normal distribution lies within two standard deviations of the mean to estimate $\Pr(Y > 124)$. 1 mark

d. Find $\mathrm{E}(X_1 - 2X_2)$ and $\mathrm{Var}(X_1 - 2X_2)$, where X_1 and X_2 are independent observations of X. 2 marks

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Question 11A.4*** (4 marks) ☐ Done ☐ Marked ☐ Redo

A fair coin is tossed 100 times. Let X be the number of heads. You may use $\Pr(Z < 2) = 0.977$.

a. State the approximate distribution of X, with its mean and standard deviation. 2 marks

b. Estimate $\Pr(X \geq 60)$, and explain why the approximation is reasonable. 2 marks

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Section B – Technology active

Show the equation you solve or the integral you evaluate.

Question 11B.1*** (6 marks) ☐ Done ☐ Marked ☐ Redo

X has probability density function $f(x) = \dfrac{x}{2}$ for $0 \leq x \leq 2$ (and 0 otherwise).

a. Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. 3 marks

b. State the approximate distribution of the mean $\bar{X}$ of a random sample of 50 values of X. 1 mark

c. Find $\Pr(\bar{X} > 1.4)$, correct to three decimal places. 2 marks

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Question 11B.2***** (15 marks) ☐ Done ☐ Marked ☐ Redo

A council claims that the mean time it takes to repair a pothole is 12 days. Repair times are normally distributed with standard deviation 4 days. A residents' group believes the mean is longer, and tests H_0: $\mu = 12$ against H_1: $\mu > 12$ at the 1% level of significance using a random sample of n repairs.

a. The group will reject H_0 if the sample mean is more than 13.163 days (correct to three decimal places). Find n. 2 marks

b. With this test, the probability of a Type II error is 0.2503. Find the true mean repair time, correct to one decimal place. 3 marks

c. The group doubles its sample size and keeps the same level of significance. Find the new critical value, correct to three decimal places, and the new probability of a Type II error, correct to four decimal places. 3 marks

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A student is told only the level of significance, the standard deviation, and that the probability of a Type II error is 0.2503 for a sample of size n and 0.0277 for a sample of size $2n$. The student is *not* told n, the critical values c_1 and c_2, or the true mean μ_T, and tries to find all four.

d. Write down the four equations the student can form. Give values from the standard normal distribution correct to four decimal places. 4 marks

e. By eliminating c_1 from one pair of equations and c_2 from the other pair, show that the four equations cannot be solved for μ_T. Give two different pairs of values of n and μ_T that fit all of the information. 3 marks

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Section C – Multiple choice (technology active)

Exam 2 style, four options. Shade one answer.

Question 11C.1** (1 mark) ☐ Done ☐ Marked ☐ Redo

A sample of 36 croissants has mean length 15 cm. The population variance is 2 cm^2. An approximate 95% confidence interval for the mean length is

A. $(14.35,\ 15.65)$ **B.** $(14.54,\ 15.46)$ **C.** $(14.67,\ 15.33)$ **D.** $(12.23,\ 17.77)$

Answer: (A) (B) (C) (D)

Question 11C.2*** (1 mark) ☐ Done ☐ Marked ☐ Redo

X has probability density function $f(x) = 2x$ for $0 \leq x \leq 1$ (and 0 otherwise). For random samples of size 50, the standard deviation of $\bar{X}$ is

A. $\dfrac{1}{50}$ **B.** $\dfrac{\sqrt{2}}{6}$ **C.** $\dfrac{1}{30}$ **D.** $\dfrac{1}{900}$

Answer: (A) (B) (C) (D)

Question 11C.3** (1 mark) ☐ Done ☐ Marked ☐ Redo

Boxes are claimed to have mean mass 400 g, with $\sigma = 10$ g. A sample of 25 boxes has mean 396 g. For $H_1 : \mu < 400$, the p-value is closest to

A. 0.0456 **B.** 0.4207 **C.** 0.9772 **D.** 0.0228

Answer: (A) (B) (C) (D)

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Probability and Statistics – Short answers

Final answers only, taken from the last line of each worked solution. Full working, and every method, is in the Worked Solutions. The grey number is the notes section the question tests: if you got it wrong, reread that section first. If your answer looks different, check the Solutions before you decide it is wrong: it may be the same answer in another form.

11A.1 [11.1] **a.** $\mathrm{E}(2X-3)=2(5)-3=7;\ \mathrm{Var}(2X-3)=2^2(4)=16$ **b.** $\mathrm{E}(X-Y)=5-3=2;\ \mathrm{Var}(X-Y)=4+9=13$ (variances add, even for a difference) **c.** see Solutions

11A.2 [11.4] **a.** $(50.04,\ 53.96)$ hours **b.** explanation in the Solutions **c.** $n \geq 384.16$, so $n=385$ **d.** Wider, because the z-value is larger (2.576 instead of 1.96)

11A.3 [11.1] **a.** $\mathrm{E}(X)=1.1,\ \mathrm{Var}(X)=0.49$ **b.** see Solutions **c.** $124=110+2\times 7$, so $\Pr(Y>124)\approx\frac{1-0.95}{2}=0.025$ **d.** $\mathrm{E}(X_1-2X_2)=1.1-2.2=-1.1;\ \mathrm{Var}(X_1-2X_2)=0.49+4(0.49)=2.45$

11A.4 [11.3] **a.** mean $np=50$, sd $\sqrt{np(1-p)}=5$ **b.** see Solutions

11B.1 [11.2] **a.** see Solutions **b.** see Solutions **c.** $\Pr(\bar{X}>1.4)=\Pr(Z>1)\approx 0.159$

11B.2 [11.6] **a.** $\sqrt{n}=\dfrac{4\times 2.3263}{1.163}=8.00$, so $n=64$ **b.** $\mu_T=13.5$ days **c.** $c_2=12.822$; $\Pr(\bar{X}<12.822\mid\mu=13.5)=0.0277$ **d.** see Solutions **e.** For example $n=64$, $\mu_T=13.5$ and $n=144$, $\mu_T=13$ (also $n=36$, $\mu_T=14$)

Section C (multiple choice): 1 B 2 C 3 D

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